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Constitutive Equations for Large Deformation, Rate-Dependent Plasticity for Metal Alloys.

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CONSTITUTIVE EQUATIONS FOR LARGE DEFORMATION, RATE DEPENDENT PLASTICITY FOR METAL ALLOYS

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To My Parents and My Wife

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ABSTRACT

A system of constitutive equations is proposed that describes the viscoplastic behavior of a class of metal alloys when subjected to large deformations. The model is purely elastic before the plastic state is reached, and becomes elastic/viscoplastic after the plastic state has been exceeded. Three main assumptions are made with regard to the strain-rate dependency in the proposed formulation. The inelastic strain rate tensor is assumed to be normal to each point of the rate dependent convex yield surface. The initial yield stress obtained from uniaxial tests is assumed to be dependent on the rate of strain loading. Finally, the hardening effect is due to isotropic and kinematic work hardening and due to the influence of the strain-rate effect.

Uniaxial tests are conducted in this work on specimens made of commercially pure aluminum in order to check the validity of the proposed constitutive model and to determine the material parameters of the model. The uniaxial loading-reverse loading tests are conducted at five different constant strain-rate values in an effort to obtain a wide range of applicability of the proposed model.

The material behavior is simulated numerically using the proposed constitutive model, and compared with the experimental results in order to check the accuracy of the proposed model.

The applicability and effectiveness of the proposed constitutive model in solving complex (i.e., and shape and any deformation) finite deformation problems is demonstrated by the numerical simulation of a moderately thick plate. Experimental verification is provided for the bending of a thick plate. Creep and relaxation behavior of the commercially pure aluminum is also investigated.

Chapter 1

INTRODUCTION

The fundamental assumptions of all theories of plasticity (rate independent) ignore the significant sensitivity of materials to the rates at which deformation or load is applied.

It is well known that in many practical problems the actual behavior of a material is governed by plastic as well as by rheologic effects. It can even be said that for many important structural materials rheologic effects are more pronounced after the plastic state has been reached.

Both sciences (plasticity and rheology) are concerned with the description of very important mechanical properties of structural materials. Each of them has created its own methods of investigation and has developed within the framework of certain assumptions which, unfortunately, cannot always be satisfied in reality. The results of rheology are confined to cases where plastic strain is of no decisive importance. On the other hand, the results of the theory of plasticity permit correct description of only such problems where the influence of rheologic effects may be considered unessential. In other words, if methods of rheology are used we should confine our considerations to the study of those states of stress that do not produce plastic flow of the material. If methods of plasticity are used we must limit ourselves to quasi-static processes the duration of which is sufficiently short, so that creep or relaxation effects do not occur. However, recent research concerning the description of dynamic properties of materials has shown that the application of the theory of plasticity, in which rheologic effects are disregarded, leads to too large discrepancies between the theoretical and experimental results [1].

However, theories of viscoplasticity make simultaneous description of the plastic and rheologic properties of the material possible. Creep and stress relaxation can also be described by the viscoplastic constitutive relations. The methods of viscoplasticity belong neither to rheology nor to plasticity.

The difficulties of combined treatment of rheologic and plastic phenomena are enormous. The viscous properties of the material introduce a time dependence of the states of stress and strain. The plastic properties, on the other hand, make these states depend on the loading path.

Thus, as a result of simultaneous introduction of viscous and plastic properties, we obtain a dependence on the load history and on the time. A description of strain in viscoplasticity will therefore involve the history of the specimen, expressed in the type of the loading process, and the time. Different results will be obtained for different loading paths and different duration of the process.

Numerous theories of viscoplastic materials of varying capability and complexity have been developed in the last 65 years. These theories in general can be divided into two broad groups: theories that use the concept of a yield condition and a yield surface from time independent plasticity, and theories that assumes that inelastic deformation can occur at all stress levels, however small. No yield surface is used in the second group of theories.

Theories Assuming a Yield Condition

The first theory of viscoplastic material behavior was proposed by Bingham [2,3] in 1919 for simple shear deformation. Bingham defined viscoplastic material as any substance that flows when the applied shear stress exceeds a critical value. This is the yield stress in simple shear, such that the rate of shear deformation is linearly proportional to the excess of the absolute value of the shear stress over the yield stress.

Hohenemser and Prager [3] generalized Bingham's relation for arbitrary states of stresses. Their formulation assumes incompressible plastic deformation, the initial yield is isotropic with no subsequent hardening, the yield is rate independent, and the deformation is isothermal.

Perzyna's [6,7,8,9] constitutive model is based on the modification of Hohenemser-Prager theory by introducing time rate dependency and work hardening into the yield function and is limited to small strains. This model allows for the total deformation to include elastic as well as inelastic deformation while the other theories assumes rigid visco-plastic materials. The initial yield condition is time-independent. The initial yield surface and all subsequent time-dependent loading surfaces are assumed to be smooth convex surfaces in the stress space. The inelastic strain rate tensor is assumed always to be directed along the normal to the initial time-independent surface and to all subsequent dynamic loading surfaces.

Naghdi and Murch [10] proposed the viscoelastic/plastic theory in which the plastic deformation is time dependent. It is limited to small strain and isothermal deformation of homogeneous initially isotropic materials. The total strain is decomposed into elastic, viscous, and plastic components. The behavior of the material in the viscoelastic range is assumed to be linear isotropic viscoelastic and the initial yield depends on the time on which the load path is traversed. This model postulates a loading surface defined by a loading function. This functions counts for plastic hardening. The convexity of the loading surfaces and normality of the strain rate for a time-dependent behavior are considered through Drucker's postulate of stability.

Chaboche [11,12,13] postulated an elastic/viscoplastic theory having features similar to the one proposed by Perzyna. It assumes homogeneous initially isotropic materials limited to small isothermal deformation, where the inelastic deformation is taken to be incompressible. The stress deviator tensor is decomposed into 'active' stress deviator and an internal state variable (i.e., 'rest stress' and a 'back stress'). The internal 'rest stress' corresponds with the stress that determines the center of the yield surface in the stress space. A loading surface is postulated in terms of the active stress deviator, and a scaler function of a second internal state variable. The scaler function corresponds to the isotropic work hardening parameter which gives a measure of the size of the elastic region bounded by the yield locus. The loading surface is smooth convex in the stress space, and the inelastic strain rate tensor is assumed to be normal to the rate-dependent loading surface. The loading function is of the von Mises type, with viscoplastic kinematic and isotropic hardening.

Eisenberg and Yen [14,15] proposed a model which is based upon Perzyna's theory and incorporates an anisotropic hardening rule that allows for yield surface distortion as well as kinematic and isotropic hardening as a result of viscoplastic deformation. The theory of Philips and Wu [16] is similar to Perzyna's theory. The normality rule is modified such that the viscoplastic strain rate tensor is assumed to be normal to the initial quasi-static yield surface but not to the dynamic surface. Perzyna, Philips and Wu theories are equivalent when isotropic expansion of the yield surface is considered and von Mises type of the yield function is assumed in Perzyna's theory.

Theories without a Yield Condition

Theories without a yield condition assumes that both elastic (recoverable) and viscoplastic (nonrecoverable) deformation components are always non-zero under all loading and unloading conditions for any non-zero value of stress. These theories assumes that at the beginning of loading from the origin the nonrecoverable strains increase gradually, being very small in the range of very small strains so that the total strain approximates linear elastic stress-strain response. For large magnitude of strains the transition to nonlinear behavior is continuous with no sharply defined yield stress.

Bodner and Partom [17,18,19,20] introduced a theory for homogeneous material that is initially isotropic. The total deformations (elastic and viscoplastic) are not separated by a yield function. However, it is assumed that the rate of deformation can be decomposed into elastic and viscoplastic parts during any loading, with the viscoplastic part being incompressible. This theory assumes the viscoplastic rate of deformation is proportional to the stress deviator (perfect plastic flow where there is no strain hardening), while the elastic rate of deformation is related to the stress rate according to the linear elastic relationship.

Miller [21] introduced a microstructural based constitutive theory for small isothermal strains, and only for uniaxial stress and strain. This theory accounts for hardening (described in microstructural terms), cyclic hardening and softening, and creep.

Krempl, Liu, and Chernockey [22,23,24] proposed a uniaxial constitutive equation for small isothermal deformation that assumes an equilibrium stress-strain curve (rate independent) with the total strain appearing explicitly in the constitutive equation. Their model contains unspecified material functions of the over stress, i.e., the stress above the equilibrium stress- strain curve. The ratio of these material functions is equal to the elastic modulus, which assures a linear elastic response for extremely fast loading. This theory has the capability of reproducing primary and secondary creep.

The theory of Valanis [25,26,27,28], which is also known as endochronic theory, introduces a scaler variable (intrinsic time) that is related to the deformation history of the material. The general form of the constitutive equations of endochronic theory are developed from thermodynamic considerations [29, 30] in which an internal variable formalism is coupled with Onsger-Prigogoine-DeGroot theory of non-equilibrium thermodynamics. This theory is limited to small strains at uniform temperature.

A brief overview of the theories of viscoplasticity that has been developed in the last 65 years has been discussed. However, only a few of these theories by Naghdi and Murch [10], Perzyna and Wojno [9], Green and Naghdi [33], and Bodner and Partom [18], were developed or extended for finite strain deformation analysis.

The constitutive equations for plasticity presented by Voyiadjis [34] and Voyiadjis and Kiousis [35] are modified here in order to incorporate rate sensitivity in the plastic region. Some of the basic concepts of the theory of viscoplasticity outlined by Naghdi and Murch [10], Perzyna and Wojno [9], and Eisenberg and Yen [14] are used in this work in order to formulate the proposed viscoplastic constitutive model for finite strain deformation analysis. The proposed model is purely elastic before the plastic state is reached, and becomes elastic/viscoplastic after the plastic state has been exceeded. Three main assumptions are made with regard to the strain-rate dependency in the proposed formulation. The inelastic strain rate tensor is assumed to be normal to each point of the rate dependent convex yield surface. The initial yield stress obtained from uniaxial tests is assumed to be dependent on the rate of strain loading. Finally, the hardening effect is due to isotropic and kinematic work hardening and due to the influence of the strain-rate effect.

Uniaxial tests are conducted in this work on specimens made of commercially pure aluminum in order to check the validity of the proposed constitutive model and to determine the material parameters of the model. The uniaxial loading-reverse loading tests are conducted at five different constant strain-rate values in an effort to obtain a wide range of applicability of the proposed model.

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Chapter 2

THEORETICAL FORMULATION

The viscoplastic model proposed in this research is formulated on the basis of the static yield criterion, associated flow rule, and hardening rules defined for the rate independent plasticity model proposed by Voyiadjis [34] and Voyiadjis and Kiousis [35]. In the following discussion, two coordinate systems are employed. Spatial or Eulerian coordinates describe the location of a point in the material using the instantaneous or deformed state as reference. These coordinates are indicated by lower case Lattin suffixes. Material or Lagrangian coordinates describe the location of a point with respect to the original (undeformed) state. These are indicated by capital Lattin suffixes.

Some of the basic concepts of the theory of viscoplasticity outlined by Naghdi and Murch [10], Perzena and Wojno [9], and Eisenberg and Yen [14] are used in this research in order to formulate the proposed viscoplastic constitutive model for finite strain deformation analysis.

The proposed model is purely elastic before the plastic state is reached and becomes elastic/viscoplastic after the plastic state has been exceeded.

A yield criterion of the von-Mises type expressed in terms of the Cauchy stress is used in this formulation. The loading function in this criterion accounts for both isotropic and kinematic hardening of the Prager-Ziegler type as proposed by Ziegler [36], and is given by

$$F = \left[\frac{\frac{1}{2}(\bar{t}_{k\ell} - \bar{\alpha}_{k\ell})(\bar{t}_{k\ell} - \bar{\alpha}_{k\ell})}{k^2 + c\,\kappa}\right]^{1/2} - 1 \tag{1}$$

where $\bar{\alpha}_{k\ell}$ is the deviator component of the shift stress tensor $\alpha_{k\ell}$ in the Eulerian reference frame, $\bar{t}_{k\ell}$ is the deviatoric component of the Cauchy stress tensor $t_{k\ell}$, k is a constant that describes the initial yield stress for which viscoplastic flow sets in first, and c is a parameter that governs the isotropic hardening. κ is the viscoplastic work done during the viscoplastic flow and is related to the Cauchy stress and the viscoplastic component $d''_{k\ell}$ of the spatial strain rate tensor $d_{k\ell}$ where

$$d_{k\ell} = d'_{k\ell} + d''_{k\ell} = \frac{1}{2}\left(\frac{\partial v_k}{\partial z_\ell} + \frac{\partial z_\ell}{\partial v_k}\right) \tag{2}$$

In equation (2), $d'_{k\ell}$ is the elastic component of the spatial strain rate tensor. In equation (2), $\underset{\sim}{v}$ and $\underset{\sim}{z}$ are the spatial descriptions of the velocity and spatial descriptions of the body respectively. In general, the decomposition of the spatial strain rate tensor d_{k1} into the "elastic" and "viscoplastic" components $d'_{k\ell}$ and $d''_{k\ell}$ respectively is purely mathematical and the components are defined by the constitutive law. However, when the elastic strains are small compared to the viscoplastic ones, a state that is satisfied in numerous applications, the above decomposition acquires the usual physical meaning. In this work, the elastic component of the strain tensor is assumed to be relatively small although the theory itself is formulated for finite strains. The rate of inelastic work is expressed as

$$\dot{\kappa} = t_{k\ell}\, d''_{k\ell} \tag{3}$$

and

$$\kappa = \int_0^t t_{k\ell}\, d''_{k\ell}\, dt \tag{4}$$

The flow rule used here to relate the viscoplastic strain rate to the stress and the stress rate in the Eulerian reference frame is of the associated type, and is expressed as

$$d''_{k\ell} = \gamma(F) \ <\phi(F)> \frac{\partial F}{\partial t_{k\ell}} \tag{5}$$

the symbol $<\phi(F)>$ is defined as follows:

$$<\phi(F)> = \begin{cases} 0 & \text{for } F \leq 0 \\ \phi(F) & \text{for } F > 0 \end{cases}$$

where $\gamma(F)$ and $\phi(F)$ are material functions determined experimentally from tests on the dynamic behavior of the material. The viscoplastic incompressibility of the material is valid in this case since the loading function F of equation (1) is a function of the deviatoric component of the Cauchy stress $\underset{\sim}{t}$ and back stress $\underset{\sim}{\alpha}$. Consequently equation (5) yields $d''_{kk} = 0$, which indicates the viscoplastic incompressibility of the material. In equation (5), the viscoplastic strain rate is assumed normal to each point of the dynamic convex loading surface.

Figure 1 shows a one-dimensional yield stress versus strain rate curve. Manjoine [37] states that this "S" shaped curve is typical of metals tested within a given range of strain rates. The particulars of the curve vary for each material type.

An approximation to this characteristics yield stress versus strain rate curve is expressed as [40]

$$\dot{e}''_{11} = a_1(F)^{-n_1} + a_2(F)^{n_2} (F_L - F)^{-n_3} \tag{6}$$

where

$$F_L = (\sigma_L/\sigma_0)^2 - 1 \tag{7}$$

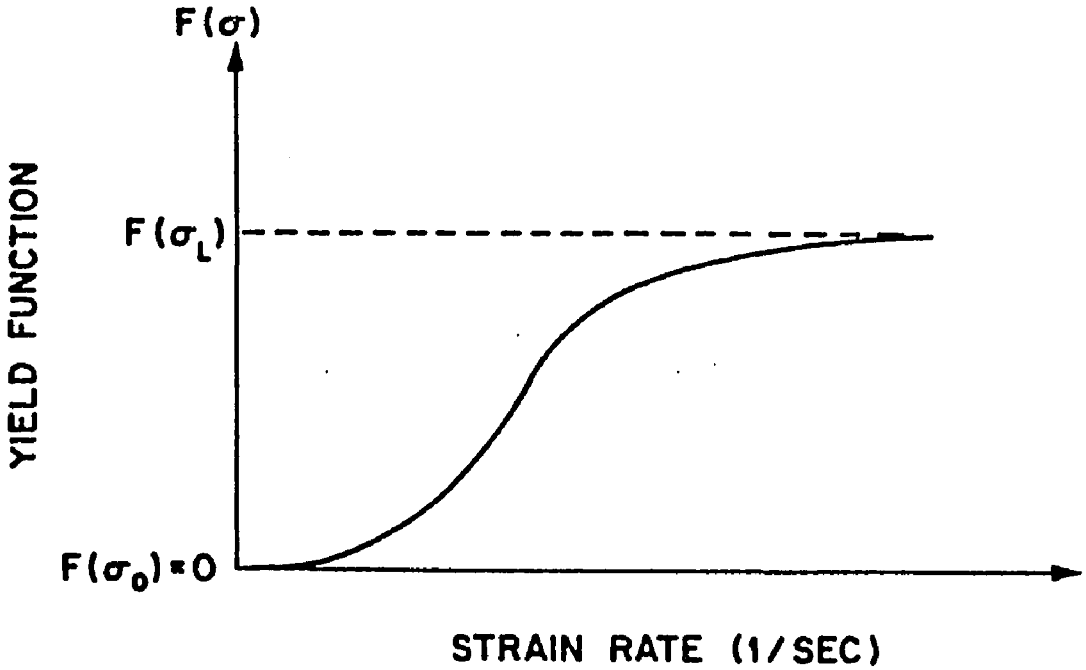

Fig. 1 Yield function versus strain rate; typical behavior for mild steel

and σ_L is a limiting one-dimensional yield stress obtained from dynamic tests at the highest strain rate of interest. $\dot{e}''_{11}$ is the viscoplastic component of the Lagrangian strain rate along the axial direction.

In this work, the material function $\gamma(F)$ is expressed as

$$\phi(F) = F \tag{8}$$

Expressing equation (5) for the case of uniaxial test, one obtains

$$d''_{11} = \gamma(F)\ F \frac{\partial F}{\partial t_{11}} \tag{9}$$

A more general expression for γ may be used for metals such that

$$\gamma(F) = a_1(F)^{-n_1}\ (F_L - F)^{-n_2} \tag{10}$$

The constants a_1, n_1 and n_2 are evaluated by curve fitting the one-dimensional stress versus strain rate experimental data.

The development of an incremental constitutive stiffness tensor, D_{ABCD}, that relates the stress rate in the Lagrangian reference frame is the primary objective in this formulation. However, the equations developed thus far are in the Eulerian reference frame. Therefore, coordinate transformations need to be applied to equations (1) and (5) to enable their use in the Lagrangian reference frame. Furthermore, a similar decomposition to that of the spatial strain tensor d_{kl} is assumed for the Lagrangian, or material, strain rate tensor $\dot{e}_{AB}$. That is

$$\dot{e}_{AB} = \dot{e}'_{AB} + \dot{e}''_{AB} \tag{11}$$

where

$$\dot{e}_{AB} = \frac{\partial e_{AB}}{\partial t} \tag{12}$$

and e'_{AB} and e''_{AB} represent the "elastic strain" and the "viscoplastic strain," respectively. In general, these two components are simply mathematical quantities defined by the constitutive law. Such an additive decomposition of the total Lagrangian strain, together with the elastic relations via the elastic modulus E_{ABCD} in the reference configuration, have been shown to be inappropriate from the point of view of capturing the physics of large deformation elastoviscoplasticity. Nevertheless, assuming small elastic deformations, but large viscoplastic strains, removes the inherent deficiency of such an approach. In this work, the added assumption is made that the elastic strains are small compared to the viscoplastic ones (an assumption satisfied in a considerable number of applications), and therefore the kinematic interpretation of the components of equation (11) acquires the usual physical meaning. A superscript dot implies material time differentiation in this text.

The relations outlined below are essential for applying coordinate transformations. The viscoplastic component $\dot{e}''_{AB}$ of the material strain rate tensor is assumed to be related to the viscoplastic component $d''_{k\ell}$ of the spatial strain rate tensor by

$$\dot{e}''_{AB} = d''_{k\ell} \frac{\partial z_k}{\partial x_A} \frac{\partial z_\ell}{\partial x_B} \tag{13}$$

In equation (13), $\underset{\sim}{x}$ is the material description of the coordinate of the body. Furthermore, (see reference [38]) we have

$$s_{AB} = t_{k\ell} \frac{\partial x_A}{\partial z_\ell} \frac{\partial x_B}{\partial z_\ell} J \tag{14}$$

where s_{AB} is the second Piola-Kirchoff stress tensor, and J is the determinant of the Jacobian of deformation. Similarly, we have

$$\alpha_{k\ell} = A_{AB} \frac{\partial z_k}{\partial x_A} \frac{\partial z_\ell}{\partial x_B} J^{-1} \tag{15}$$

where

$$A_{AB} = \int_0^t \dot{A}_{AB}\, dt \tag{16}$$

and

$$\dot{A}_{AB} = \frac{\partial A_{AB}}{\partial t} \tag{17}$$

A_{AB} is the equivalent Langrangian counterpart of the spatial shift stress tensor $\alpha_{k\ell}$. Finally, we know that

$$C_{AB} = \frac{\partial z_k}{\partial x_A} \frac{\partial z_k}{\partial x_B} \tag{18}$$

where C_{AB} is known as Green's deformation tensor.

Equation (1) may now be expressed in the Langrangian referred frame as follows:

$$\begin{aligned} F = \{[\tfrac{1}{2} (&s_{AB}\, s_{CD}\, C_{AC}\, C_{BD}\, J^{-2} - \tfrac{1}{3}\, s_{AB}\, s_{CD}\, C_{AB}\, C_{CD}\, J^{-2}) \\ &- s_{AB}\, A_{CD}\, C_{AC}\, C_{BD}\, J^{-2} + \tfrac{1}{3}\, s_{AB}\, A_{CD}\, C_{AB}\, C_{CD}\, J^{-2} \\ &+ \tfrac{1}{2}\, (A_{AB}\, A_{CD}\, C_{AC}\, C_{BD}\, J^{-2} - \tfrac{1}{3}\, A_{AB}\, A_{CD}\, C_{AB}\, C_{DC}\, J^{-2})]/ \\ &(k^2 + c\, \kappa)\}^{1/2} - 1 \end{aligned} \tag{19}$$

The viscoplastic strain rate in the Lagrangian reference frame is expressed as

$$\dot{e}''_{AB} = J\, \gamma(F)\, \langle\phi(F)\rangle\, \frac{\partial F}{\partial s_{AB}}. \tag{20}$$

We should note that the viscoplastic incompressibility is expressed by $d''_{kk} = 0$, and not $\dot{e}''_{AA} = 0$.

Following Ziegler [36], we assume that the yield surface moves in the direction of the radius connecting the center of the yield surface with the point representing the instantaneous state of stress on the current yield surface (Figure 2). Consequently, the hardening rule becomes

$$\dot{A}_{AB} = (s_{AB} - A_{AB})\, \dot{\mu} \tag{21}$$

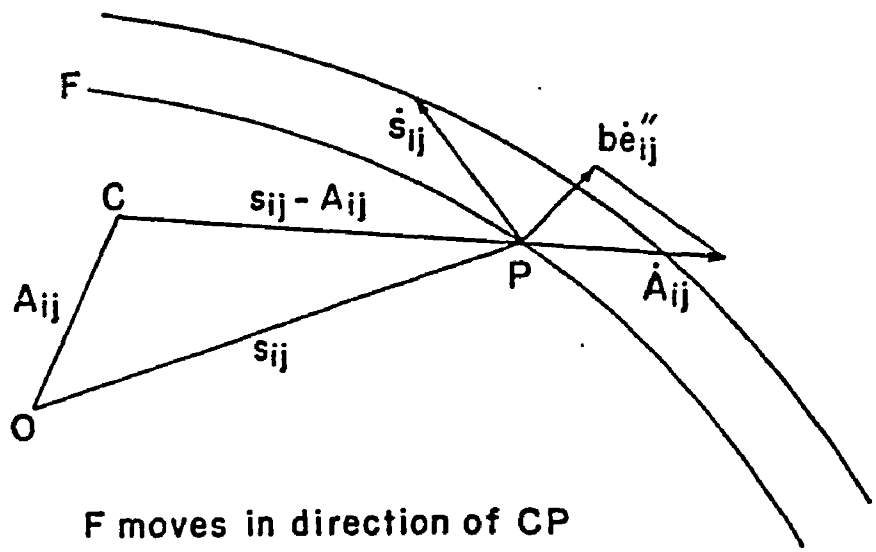

Figure 2. Modification of Prager's Hardening Rule

where $\dot{\mu}$ is a scalar function and is positive for loading as well as for reverse-loading. It is determined by noting that the projection of $\dot{A}_{AB}$ on the stress gradient of the yield surface can be equated to $b\dot{e}''_{AB}$ (Figure 2) where b is a material parameter. The procedure to obtain $\dot{\mu}$ is outlined below.

$$b\dot{\underset{\sim}{e}}'' = (\dot{\underset{\sim}{A}} - \underset{\sim}{n})\,\underset{\sim}{n} \tag{22}$$

where $\dot{\underset{\sim}{A}}$ is the material rate of shifting of the yield surface and $\underset{\sim}{A}$ is the total translation of the yield surface in the second Piola-Kirchhoff stress space when the monotonically increasing loading or reverse-loading is completed. $\underset{\sim}{n}$ is the normal unit vector to the yield surface at the stress point P (Figure 2). Using indicial notation, we obtain

$$n_{CD} = \frac{\dfrac{\partial F}{\partial s_{CD}}}{\left(\dfrac{\partial F}{\partial s_{MN}}\,\dfrac{\partial F}{\partial s_{MN}}\right)^{1/2}} \tag{23}$$

Therefore, from equation (22), the following is derived:

$$b\dot{e}''_{AB} = \dot{A}_{CD}\,\frac{\dfrac{\partial F}{\partial s_{CD}}\,\dfrac{\partial F}{\partial s_{AB}}}{\dfrac{\partial F}{\partial s_{MN}}\,\dfrac{\partial F}{\partial s_{MN}}} \tag{24}$$

Equations (21) and (24) give

$$\dot{e}''_{AB} = \frac{1}{b}\,(s_{CD} - A_{CD})\;\dot{\mu}\,\frac{\dfrac{\partial F}{\partial s_{CD}}\,\dfrac{\partial F}{\partial s_{AB}}}{\dfrac{\partial F}{\partial s_{MN}}\,\dfrac{\partial F}{\partial s_{MN}}} \tag{25}$$

Substituting for $\dot{e}''_{AB}$ from (20) into (25) and solving for $\dot{\mu}$ yields

$$\dot{\mu} = J\gamma F b\,\frac{\dfrac{\partial F}{\partial s_{MN}}\,\dfrac{\partial F}{\partial s_{MN}}}{(s_{CD} - A_{CD})\,\dfrac{\partial F}{\partial s_{CD}}} \tag{26}$$

The rate of inelastic work in equation (3) is expressed in the Lagrangian coordinate system as

$$\dot{\kappa} = s_{AB}\, \dot{e}''_{AB}/J \tag{27}$$

The tangent inelastic stiffness is first formulated by ignoring second order terms in the increment of time and using a generalization approach of the work by Kanchi, Zienkiewicz and Owen [39].

From equation (18), the visco-plastic material strain rate is presumed to be a function of the following

$$\dot{e}''_{AB} = \beta_{AB}(s_{PQ},\ J,\ e_{PQ},\ \kappa,\ A_{PQ}) \tag{28}$$

We define a strain increment $\Delta e''^{(n)}_{AB}$ occurring at a time interval

$$\Delta t^{(n)} = t^{(n+1)} - t^{(n)} \tag{29}$$

using the implicit scheme

$$\Delta e''^{(n)}_{AB} = \Delta t^{(n)}\, [(1 - \theta)\, \dot{e}''^{(n)}_{AB} + \theta\, \dot{e}''^{(n+1)}_{AB}] \tag{30}$$

The time interval $\Delta t^{(n)}$ denotes the n plus first time step. In equation (30), setting θ equal to zero we obtain the fully explicit Euler scheme, while setting θ equal to one we get the fully implicit scheme. The trapezoidal scheme is obtained by setting θ equal to one-half.

The increments of J, κ, $\underset{\sim}{e}$ and $\underset{\sim}{A}$ are defined similarly as

$$\begin{aligned}
\Delta J^{(n)} &= \Delta t^{(n)}\, [(1 - \theta)\, \dot{J}^{(n)} + \theta\, \dot{J}^{(n+1)}] \\
\Delta \kappa^{(n)} &= \Delta t^{(n)}\, [(1 - \theta)\, \dot{\kappa}^{(n)} + \theta\, \dot{\kappa}^{(n+1)}] \\
\Delta e^{(n)}_{AB} &= \Delta t^{(n)}\, [(1 - \theta)\, \dot{e}^{(n)}_{AB} + \theta\, \dot{e}^{(n+1)}_{AB}] \\
\Delta A^{(n)}_{AB} &= \Delta t^{(n)}\, [(1 - \theta)\, \dot{A}^{(n)}_{AB} + \theta\, \dot{A}^{(n+1)}_{AB}]
\end{aligned} \tag{31}$$

Using a truncated Taylor's series expansion, we define $\dot{e}''^{(n+1)}_{AB}$ and $\dot{J}^{(n+1)}$ in equations (30) and (31), respectively, as

$$\dot{e}''_{AB}{}^{(n+1)} = \dot{e}''_{AB}{}^{(n)} + \left(\frac{\partial \dot{e}''_{AB}}{\partial s_{PQ}}\right)^{(n)} \Delta s_{PQ}{}^{(n)} + \left(\frac{\partial \dot{e}''_{AB}}{\partial J}\right)^{(n)} \Delta J^{(n)}$$

$$+ \left(\frac{\partial \dot{e}''_{AB}}{\partial e_{PQ}}\right)^{(n)} \Delta e_{PQ}{}^{(n)} + \left(\frac{\partial \dot{e}''_{AB}}{\partial \kappa}\right)^{(n)} \Delta \kappa^{(n)}$$

$$+ \left(\frac{\partial \dot{e}''_{AB}}{\partial A_{PQ}}\right) \Delta A_{PQ}{}^{(n)} \tag{32}$$

and

$$\dot{J}^{(n+1)} = \dot{J}^{(n)} + \left(\frac{\partial \dot{J}}{\partial s_{PQ}}\right)^{(n)} \Delta s_{PQ}{}^{(n)} + \left(\frac{\partial \dot{J}}{\partial e''_{PQ}}\right)^{(n)} \Delta e''_{PQ}{}^{(n)}$$

$$+ \left(\frac{\partial \dot{J}}{\partial e_{PQ}}\right)^{(n)} \Delta e_{PQ}{}^{(n)} + \left(\frac{\partial \dot{J}}{\partial \kappa}\right)^{(n)} \Delta \kappa^{(n)}$$

$$+ \left(\frac{\partial \dot{J}}{\partial A_{PQ}}\right)^{(n)} \Delta A_{PQ}{}^{(n)} \tag{33}$$

Similarly, we obtain expressions for $\dot{e}_{AB}{}^{(n+1)}$, $\dot{\kappa}^{(n+1)}$ and $\dot{A}_{AB}{}^{(n+1)}$. We substitute for $\dot{e}''_{AB}{}^{(n+1)}$ from expression (32) into equation (30), to obtain

$$\Delta e''_{AB}{}^{(n)} = \Delta t^{(n)} \{\dot{e}''_{AB}{}^{(n)} + \theta \; [(\frac{\partial \dot{e}''_{AB}}{\partial s_{CD}})^{(n)} \Delta s_{CD}{}^{(n)}$$

$$+ \left(\frac{\partial \dot{e}''_{AB}}{\partial J}\right)^{(n)} \Delta J^{(n)} + \left(\frac{\partial \dot{e}''_{AB}}{\partial e_{CD}}\right)^{(n)} \Delta e_{CD}{}^{(n)}$$

$$+ \left(\frac{\partial \dot{e}''_{AB}}{\partial \kappa}\right)^{(n)} \Delta \kappa^{(n)} + \left(\frac{\partial \dot{e}''_{AB}}{\partial A_{CD}}\right)^{(n)} \Delta A_{PQ}{}^{(n)}]\} \tag{34}$$

Substituting for $\Delta J^{(n)}$, $\Delta e_{CD}{}^{(n)}$, $\Delta \kappa^{(n)}$ and $\Delta A_{PQ}{}^{(n)}$ and using a truncated Taylor's series expansions similar to equation (33) and neglecting terms of the order $[\Delta t^{(n)}]^2$ we obtain

$$\Delta e''^{(n)}_{AB} = \Delta t^{(n)} \; [\dot{e}''^{(n)}_{AB} + \theta \; (\frac{\partial \dot{e}''_{AB}}{\partial s_{PQ}})^{(n)} \; \Delta s_{PQ}{}^{(n)}] \tag{35}$$

Assuming a linear elastic relation between the second Piola-Kirchhoff stress tensor and the material strain tensor, we obtain

$$s_{AB} = E_{ABCD} \; e'_{CD} \tag{36}$$

where

$$E_{ABCD} = \lambda \; \delta_{AB} \; \delta_{CD} + G(\delta_{AC} \; \delta_{BC} + \delta_{AD} \; \delta_{BC}) \tag{37}$$

In equation (37), λ and G are Lame's constants, and $\underset{\sim}{E}$ in is the modulus of elasticity. Equation (37) may be expressed in incremental form as follows:

$$\dot{s}_{AB} = E_{ABCD} \; \dot{e}'_{CD} \tag{38}$$

or

$$\Delta s_{AB}{}^{(n)} = E_{ABCD} \; \Delta e'^{(n)}_{CD} \tag{39}$$

Making use of (12) together with equation (38), we obtain

$$\Delta s_{AB}{}^{(n)} = E_{ABCD} \; (\Delta e_{CD}{}^{(n)} - \Delta e''^{(n)}_{CD}) \tag{40}$$

We now substitute for $\Delta e''^{(n)}_{CD}$ from equation (35) into expression (40) to obtain

$$\Delta s_{AB}{}^{(n)} = E_{ABCD} \; [\Delta e_{CD}{}^{(n)} - \Delta t^{(n)} \; \dot{e}''^{(n)}_{CD}$$

$$- \Delta t^{(n)} \; \theta \; (\frac{\partial \dot{e}''_{CD}}{\partial s_{PQ}})^{(n)} \; \Delta s_{PQ}{}^{(n)}] \tag{41}$$

or

$$\Delta s_{PQ}{}^{(n)} \; [\delta_{PA} \; \delta_{QB} + \theta \; \Delta t^{(n)} \; (\frac{\partial \dot{e}''_{CD}}{\partial s_{PQ}})^{(n)} \; E_{ABCD}] =$$

$$E_{ABMN} \; [\Delta e_{MN}{}^{(n)} - \Delta t^{(n)} \; \dot{e}''^{(n)}_{MN}] \tag{42}$$

Let $\underset{\sim}{N}^{(n)}$ be the inverse of the tensor $\underset{\sim}{M}^{(n)}$, where

$$M_{ABPQ} = \delta_{PA}\ \delta_{QB} + \theta\ \Delta t^{(n)}\ \left(\frac{\partial \dot{e}''_{CD}}{\partial s_{PQ}}\right)^{(n)}\ E_{ABCD} \tag{43}$$

From equation (42), we obtain

$$\Delta s_{RS}{}^{(n)} = D_{RSPQ}{}^{(n)}\ \Delta e_{PQ}{}^{(n)} - D_{RSPQ}{}^{(n)}\ \Delta t^{(n)}\ \dot{e}''_{PQ}{}^{(n)} \tag{44}$$

where

$$D_{RSPQ}{}^{(n)} = N_{RSAB}{}^{(n)}\ E_{ABPQ} \tag{45}$$

$\underset{\sim}{D}^{(n)}$ will be termed the elasto/visco-plastic tangent modulus.

Chapter 3

DETERMINATION OF MATERIAL PARAMETERS

A number of uniaxial loading-reverse loading tests at constant strain rates are performed on specimens made of commercially pure aluminum in order to check the validity of the proposed model and to determine the material parameters appearing in the analytical formulation.

The uniaxial tests were performed on specimens of circular cross section, and their dimensions are shown in Figure 3. The specimens were tested using a servocontrolled MTS* testing machine. The uniformity of the wall thickness of the test section is to within $\pm 5 \times 10^{-4}$ inch for all the specimens. The specimens are machined to their appropriate dimensions from a cold-rolled aluminum bar 1.0 inch in diameter. The strain was measured by a clip-on extensometer.

The specimens are first subjected to uniform tension up to a load that produces about six percent strain. The load is then reversed and the specimens are uniaxially compressed up to a load that causes plastic deformations without buckling the specimen.

In Figure 4 the experimental results from uniaxial loading-reverse loading tests are shown for the commercially pure aluminum specimens. The uniaxial tests are conducted at five different constant spatial strain rates d_{11} ranging from 1.8×10^{-5}/second to 10^{-1}/second. Different strain rates are used in some cases for each of the loading and reverse loading paths. For specimen A, during loading a strain rate of 1.8×10^{-5}/second is used, while during reverse loading of the same

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*MTS Systems Corporation, Minneapolis, MN, USA.

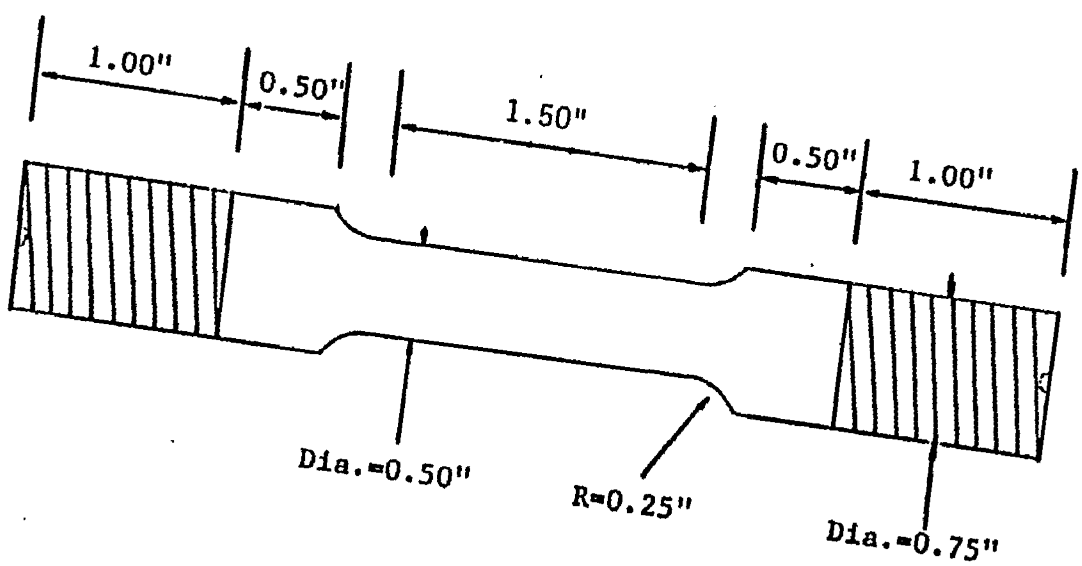

Figure 3. Dimensions of the Specimen Used

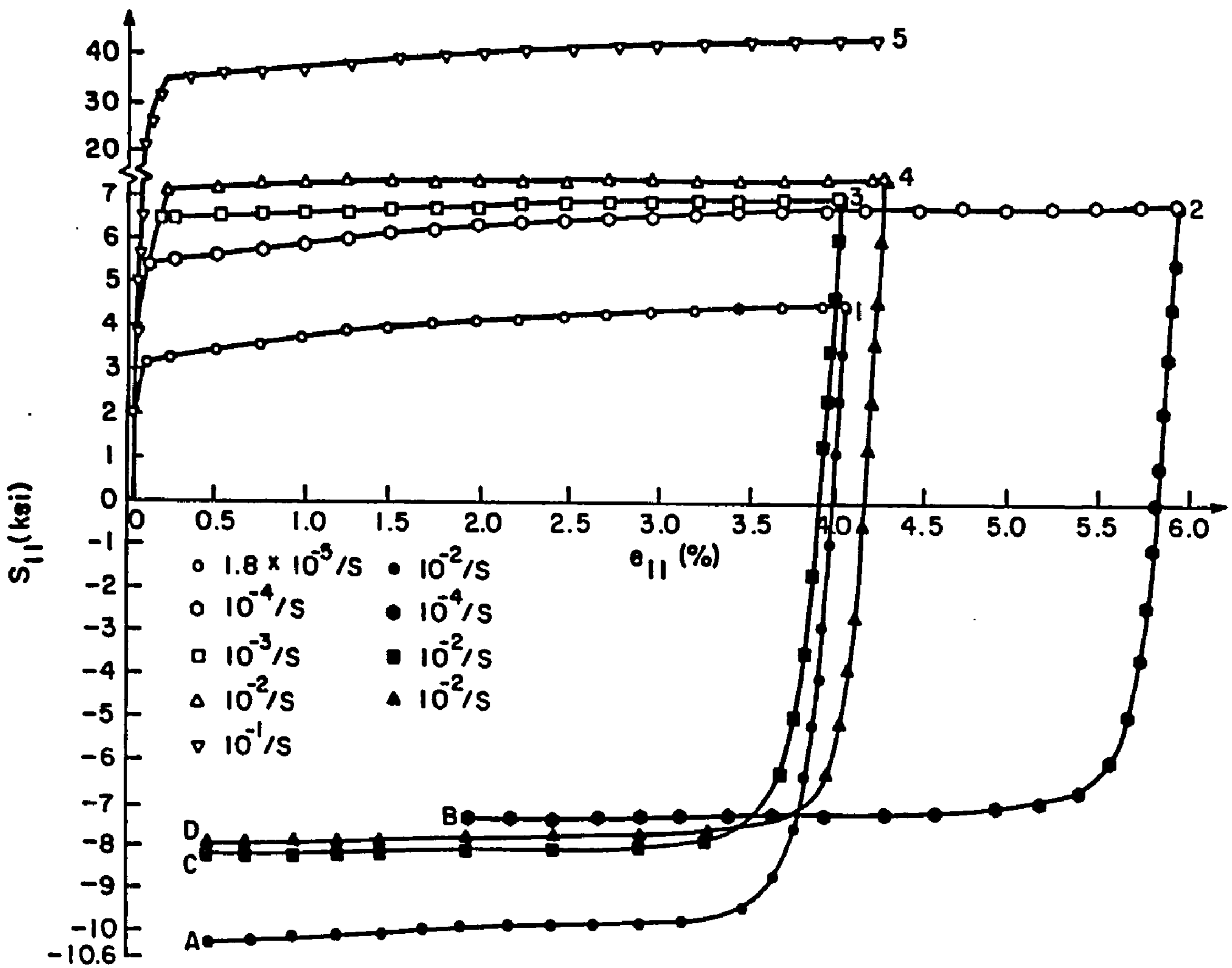

FIGURE 4. EXPERIMENTAL RELATION BETWEEN MATERIAL STRAIN e_{11} AND SECOND PIOLA-KIRCHHOFF STRESS S_{11}

specimen, a strain rate of 10^{-2}/second is used. The following elastic material parameters are obtained from the experimental results:

Young's Modulus = 10,400 ksi

Poisson's ratio = 0.25

Different types of hardening of the material undergoing viscoplastic deformation are considered separately in the following discussion. A method of obtaining the appropriate material parameters for each case is also outlined below.

In the case of uniaxial loading and reverse-loading, the material stress rate shift tensor $\dot{A}_{CD}$ in equation (24) and the second Piola-Kirchhoff stress rate $\dot{s}_{11}$ have the same direction in the stress space. Therefore, $\dot{A}_{CD}$ becomes $\dot{A}_{11}$ and the multiaxial equation (24) reduces to

$$b\ \dot{e}''_{11} = \dot{A}_{11} \frac{\left(\frac{\partial F}{\partial s_{11}}\right)^2}{\left(\frac{\partial F}{\partial s_{MN}}\right)\left(\frac{\partial F}{\partial s_{MN}}\right)} \tag{46}$$

The absence of plastic volumetric strains is assumed in the formulation presented in this work, and is expressed mathematically as

$$d''_{kk} = 0 \tag{47}$$

Substituting for d''_{kk} from (13) into (47) yields

$$\dot{e}''_{AB} \frac{\partial x_A}{\partial z_k} \frac{\partial x_B}{\partial z_k} = 0 \tag{48}$$

Making use of equation (18), equation (48) can be expressed as

$$\dot{e}''_{AB}\ C^{-1}_{AB} = 0 \tag{49}$$

Differentiating the expression for the yield function in equation (19) with respect to the second Piola-Kirchhoff stress s_{MN} gives

$$\frac{\partial F}{\partial s_{MN}} = \frac{s_{OP}C_{OM}C_{PN} - \frac{1}{3}\,s_{OP}C_{OP}C_{MN} - A_{OP}C_{MO}C_{NP} + \frac{1}{3}\,A_{OP}C_{OP}C_{MN}}{2\ J^2\ [J_2^*]^{1/2}\ [k^2 + c\kappa]^{1/2}} \tag{50}$$

where

$$J_2^* = \frac{1}{J^2} [\frac{1}{2} (s_{AB}\ s_{CD}\ C_{AC}\ C_{BD} - \frac{1}{3}\ s_{AB}\ s_{CD}\ C_{AB}\ C_{CE})$$

$$- s_{AB}\ A_{CD}\ C_{AC}\ C_{BD} + \frac{1}{3}\ s_{AB}\ A_{CD}\ C_{AB}\ C_{CD}$$

$$+ \frac{1}{2} (A_{AB}\ A_{CD}\ C_{AC}\ C_{BD} - \frac{1}{3}\ A_{AB}\ A_{CD}\ C_{AB}\ C_{CD})] \tag{51}$$

Since in a uniaxial test only s_{11} and A_{11} are nonzero, equation (50) simplifies to

$$\frac{\partial F}{\partial s_{MN}} = \frac{s_{11} C_{M1} C_{No} - \frac{1}{3}\ s_{11} C_{11} C_{MN} - A_{11} C_{M1} C_{N1} + \frac{1}{3}\ A_{11} C_{11} C_{MN}}{2\ J^2\ [J_2^*]^{1/2}\ [k^2 + c\kappa]^{1/2}} \tag{52}$$

Similar simplification is made to equation (51).

Now Green's deformation tensor C_{MN} is related to the material strain tensor e_{MN} through the following relationship:

$$C_{MN} = 2\ e_{MN} + \delta_{MN} \tag{53}$$

Furthermore, in a uniaxial test, C_{MN} is nonzero only when M = N since e_{MN} is nonzero if and only if M = N. Therefore, it follows from equation (52) that it is nonzero only if M = N. This yields

$$\frac{\partial F}{\partial s_{MN}} \frac{\partial F}{\partial s_{MN}} = (\frac{\partial F}{\partial s_{11}})^2 + (\frac{\partial F}{\partial s_{22}})^2 + (\frac{\partial F}{\partial s_{33}})^2 \tag{54}$$

Equation (20) can now be utilized to express equation (54), and therefore, equation (46) in terms of the material viscoplastic strain rate tensor. However, before proceeding with that, it is worth noting that the axi-symmetric nature of the uniaxial test required that

$$\frac{\partial F}{\partial s_{22}} = \frac{\partial F}{\partial s_{33}} \tag{55}$$

Equation (55) follows from the expression (20) since the axisymmetric nature of the problem requires that $\dot{e}''_{22}$ be equal to $\dot{e}''_{33}$.

Substituting from (54), (55) and (20) into equation (46) yields

$$b\,\dot{e}''_{11} = \dot{A}_{11}\,\frac{(\dot{e}''_{11})^2}{(\dot{e}''_{11})^2 + 2(\dot{e}''_{22})^2} \tag{56}$$

Also, for a uniaxial test, the expression in (49) simplifies to

$$\dot{e}''_{11}\,C^{-1}_{11} + \dot{e}''_{22}\,C^{-1}_{22} + \dot{e}''_{33}\,C^{-1}_{33} = 0 \tag{57}$$

Therefore, from axi-symmetry, we obtain

$$\dot{e}''_{22} = -\,\frac{C^{-1}_{11}}{2\,C^{-1}_{22}}\,\dot{e}''_{11} = -\,\frac{C_{22}}{2\,C_{11}}\,\dot{e}''_{11} \tag{58}$$

The substitution of $\dot{e}''_{22}$ from (58) into (56) gives

$$b\,\dot{e}''_{11} = \dot{A}_{11}\,\frac{1}{1 + \frac{1}{2}\,(\frac{C_{22}}{C_{11}})^2} \tag{59}$$

From (53), we have

$$C_{11} = 2\,e_{11} + 1 \tag{60}$$

and

$$C_{22} = 2\,e_{22} + 1 \tag{61}$$

Substituting for C_{11} and C_{22} from (60) and (61) respectively into (59) and rearranging yields the final expression used in evaluating the material parameter b. That is

$$b = (\frac{\dot{A}_{11}}{\dot{e}''_{11}})\;[\frac{2\,(2\,e_{11} + 1)^2}{2\,(2\,e_{11} + 1)^2 + (2\,e_{22} + 1)^2}] \tag{62}$$

The parameter b in equation (62) is obviously related to the kinematic hardening of the material. This is because the evaluation of b depends on $\dot{A}_{11}$ which is purely governed by the translation of the yield surface in the stress space; a phenomenon that exists in kinematic hardening only. The parameter c that appears in equations (1) and (19) describes the isotropic hardening phenomenon. This is due to the fact that it governs the expansion of the yield surface in the stress space due to the increasing viscoplastic work done in deforming the material plastically. Three different cases of hardening of the material are presented below, and the material parameters b and c are evaluated for each case.

Kinematic Hardening

For this case, the yield is assumed to undergo translation in the stress space while retaining its shape and size. Therefore, since the yield surface does not expand, we obtain

$$c = 0 \tag{63}$$

As was stated earlier for a uniaxial state of stress, the Lagrangian stress rate shift tensor $\dot{A}_{AB}$ and the Lagrangian stress rate $\dot{s}_{AB}$ have the same direction. However, in case of kinematic hardening, they are also equal (see figure 5). That is, $\dot{A}_{11} = \dot{s}_{11}$, or from Figure 5, we obtain

$$\Delta A_{11} = s_{11(n+1)} - s_{11(n)} \tag{64}$$

where Δ designates an increment, $s_{11(n+1)}$ is the yield stress at the end of the stress increment, and $s_{11(n)}$ is the yield stress at the beginning of the stress increment. The super bar on the stresses shown in Figure 5 and other figures designates compression. Note that $s_{11(n+1)}$ and $s_{11(n)}$ can be obtained directly from the experimental data for two consecutive stresses in the plastic range. As for $\dot{e}''_{11}$ that appears in the expression for b in (62), we can consider $\Delta e''_{11}$ as follows:

$$\Delta e''_{11} = e''_{11(n+1)} - e''_{11(n)} \tag{65}$$

where $e''_{11(n+1)}$ and $e''_{11(n)}$ are the viscoplastic material strains corresponding to $s_{11(n+1)}$ and $s_{11(n)}$ respectively. However, since $e''_{11(n+1)}$ and $e''_{11(n)}$ are not readily available from the experimental

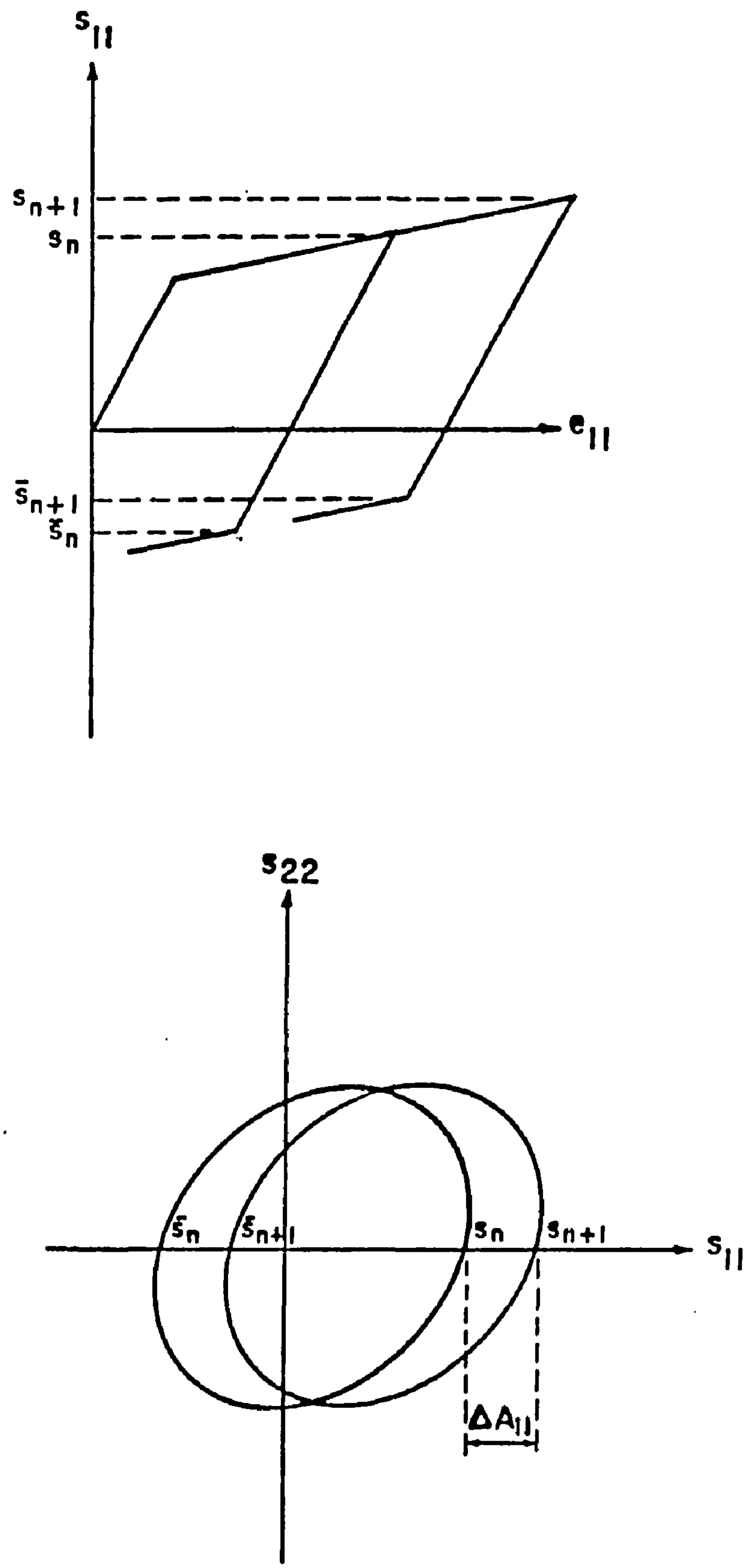

Figure 5. Case of Kinematic Hardening

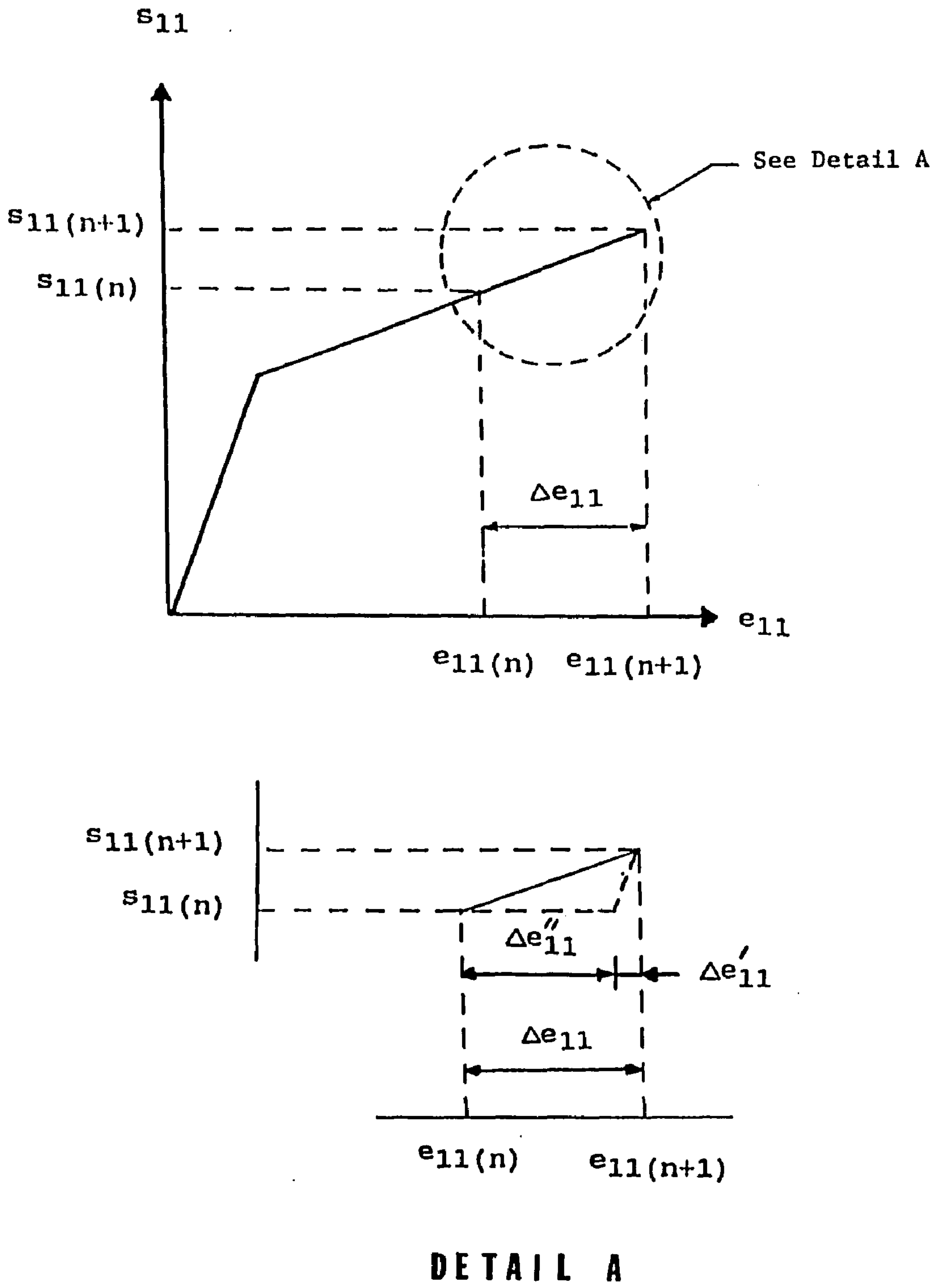

Figure 6. Experimental Determination of $\Delta e''_{11}$ for the Kinematic Hardening

data, obtaining e''_{11} can be accomplished by considering Figure 6. Hence we have

$$\Delta e_{11} = e_{11(n+1)} - e_{11(n)} \tag{66}$$

where Δe_{11} is the increment of material strain corresponding to the increment of Lagrangian stress in the viscoplastic range. $e_{11(n+1)}$ and $e_{11(n)}$ are the material strains corresponding to $s_{11(n+1)}$ and $s_{11(n)}$ respectively. $e_{11(n+1)}$ and $e_{11(n)}$ can be obtained directly from the experimental data, and upon substitution of their values in equation (55), Δe_{11} is defined. Also, from Figure 6, we obtain

$$\Delta e''_{11} = \Delta e_{11} - \Delta e'_{11} \tag{67}$$

where $\Delta e'_{11}$ is the elastic component of the material strain increment Δe_{11}. $\Delta e'_{11}$ can be obtained from the elastic response of the material after unloading occurred from the stress $s_{11(n+1)}$ to the stress $s_{11(n)}$. That is

$$\Delta e'_{11} = \frac{s_{11(n+1)} - s_{11(n)}}{E} \tag{68}$$

where E is Young's modulus of elasticity. Hence, $\Delta e''_{11}$ can now be obtained by substituting for Δe_{11} and $\Delta e'_{11}$ from (66) and (68) respectively into (67). This yields

$$\Delta e''_{11} = \frac{E[e_{11(n+1)} - e_{11(n)}] - [s_{11(n+1)} - s_{11(n)}]}{E} \tag{69}$$

It is important to realize that the expression for $\Delta e'_{11}$ in (68) is somewhat approximate since it does not account for the observed reduction in the elastic stiffness of the material undergoing finite strains. The approximation is due to the decrease in the value of E, which appears in equations (68) and (69), with increased finite strains. Nevertheless, when the decrease in the elastic stiffness of the material is not considerable, which is the case in most metals, its effect on evaluating the material parameters is insignificant. The material parameter b can now be obtained from the expression in (62) by noting that

$$\dot{A}_{11} = \frac{\Delta A_{11}}{\Delta t} \tag{70}$$

and

$$\dot{e}''_{11} = \frac{\Delta e''_{11}}{\Delta t} \tag{71}$$

where Δt represents a small time increment. Therefore, the substitution of ΔA_{11} into (70) and $\Delta e''_{11}$ into (71) from (64) and (69) respectively, and then the substitution of A_{11} and e''_{11} from (70) and (71) into (62) yields the final expression for the parameter b. Therefore, we obtain

$$b = \frac{E[s_{11(n+1)} - s_{11(n)}]}{E[e_{11(n+1)} - e_{11(n)}] - [s_{11(n+1)} - s_{11(n)}]}$$

$$\cdot \frac{2\ (2\ e_{11} + 1)^2}{2\ (2\ e_{11} + 1)^2 + (2\ e_{22} + 1)^2} \tag{72}$$

where e_{11} and e_{22} are taken to be at the beginning of the stress increment under consideration.

The expression for the parameter b represented by (72) can be used to evaluate b in the case of kinematic hardening. Equation (72) is used for several consecutive increments of stresses and strains to obtain several values of b. The results are then averaged to determine a more realistic value of the material parameter b.

Isotropic Hardening

When pure isotropic hardening is considered, the Lagrangian stress rate shift tensor $\dot{A}_{AB}$ vanishes. This is because the center of the yield surface, which initially coincides with the origin of the stress space, does not undergo translation in the process of viscoplastic deformation as shown in Figure 7. Therefore, it follows from equation (62) that

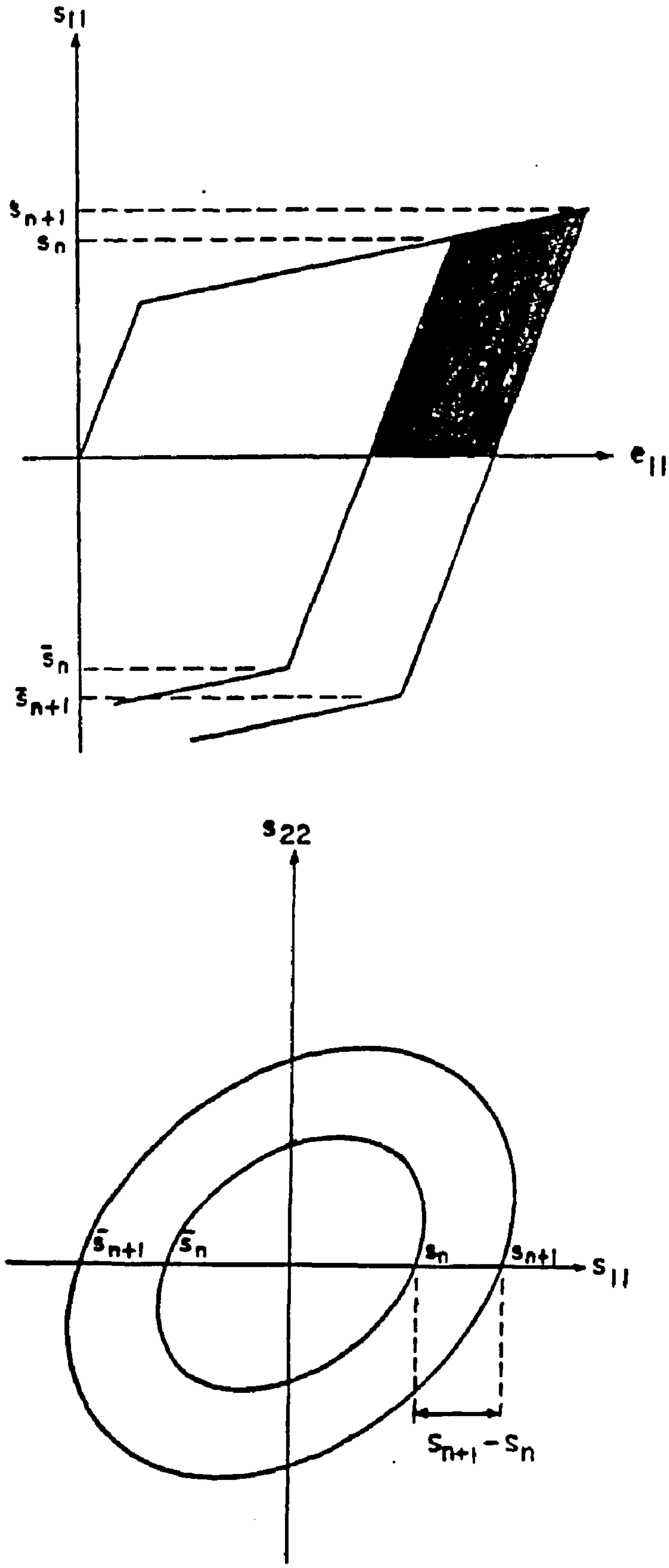

Figure 7. Case of Isotropic Hardening

$$b = 0 \tag{73}$$

It also follows that the yield function expressed in (19) reduces to

$$f \equiv [\frac{1}{2}\ [s_{11}s_{11}C_{11}C_{11}J^{-2} - \frac{1}{3}\ s_{11}s_{11}C_{11}C_{11}J^{-2}]/(k^2 + c\kappa)]^{1/2} - 1$$

which simplifies to

$$f \equiv [[\frac{1}{3}\ (s_{11})^2\ (C_{11})^2\ J^{-2}]/(k^2 + c\kappa)]^{1/2} - 1 \tag{74}$$

When the material is in a plastic state of stress, $f = 0$. Furthermore, the consistency condition asserts that loading from a plastic state leads to another plastic state. Therefore, if the state of stress in the plastic range is increased from $s_{11(n)}$ to $s_{11(n+1)}$ the following is obtained upon substitution into (74):

$$\frac{1}{3}\ [s_{11(n)}]^2\ [C_{11(n)}]^2\ [J_{(n)}]^{-2} - k^2 - c[\kappa_{(n)}] = 0 \tag{75}$$

and

$$\frac{1}{3}\ [s_{11(n+1)}]^2\ [C_{11(n+1)}]^2\ [J_{(n+1)}]^{-2} - k^2 - c[\kappa_{(n+1)}] = 0 \tag{76}$$

where the quantities with the subscript (n) are evaluated at the beginning of the stress increment, and those with (n+1) are evaluated at the end of the stress increment. Subtracting (75) from (76) and solving for the parameter c gives

$$c = \frac{[s_{11(n+1)}]^2[C_{11(n+1)}]^2[J_{(n+1)}]^{-2} - [s_{11(n)}]^2[C_{11(n)}]^2[J_{(n)}]^2}{3\Delta\kappa} \tag{77}$$

where

$$\Delta\kappa = \kappa_{(n+1)} - \kappa_{(n)} \tag{78}$$

$\Delta\kappa$ is the increment of plastic work and is equal to the area of the shaded region in Figure 7. In the final expression for the parameter c represented by (77), C_{11} can be obtained from equation (53) in this chapter. Also, the determinant of the Jacobian of deformation, J, can be expressed in terms of the Lagrangian strain invariants and evaluated accordingly. Equation (77) is used to obtain several values of the parameter c for several consecutive increments of stresses and strains. The results are then averaged to obtain a more realistic value of the material parameter c.

Combined Isotropic and Kinematic Hardening

This case of hardening is best demonstrated and explained by referring to Figure 8. In the combined isotropic and kinematic hardening, the yield surface expands as well as translates in the stress space during the state of viscoplastic flow. The expansion and translation of the yield surface are considered simultaneous as shown in Figure 8 for a uniaxial state of stress. When the material undergoing viscoplastic deformation is loaded from a viscoplastic state of stress $s_{11(n)}$ to another visoplastic state of stress $s_{11(n+2)}$, there exists a transition viscoplastic state of stress $s_{11(n+1)}$ for which the difference of $s_{11(n+1)}$ and $s_{11(n)}$ represents the expansion of the yield surface. The translation, or shift, that the yield surface undergoes in the stress space is, therefore, represented by the difference of $s_{11(n+2)}$ and $s_{11(n+1)}$. But, the translation, or shift, of the yield surface is equal to and in the direction of ΔA_{11} for a uniaxial state of stress. Hence the following is obtained:

$$s_{11(n+2)} - s_{11(n+1)} = \Delta A_{11} \tag{79}$$

Similarly, it follows from Figure 8 that

$$\bar{s}_{11(n+2)} - \bar{s}_{11(n+1)} = \Delta A_{11} \tag{80}$$

where the super bar on the stresses denotes compression. It is clear from equations (79) and (80) that although the stress rate tensor and the stress rate shift tensor continue to have the same direction in the case of combined hardening, they are not equal. This is because

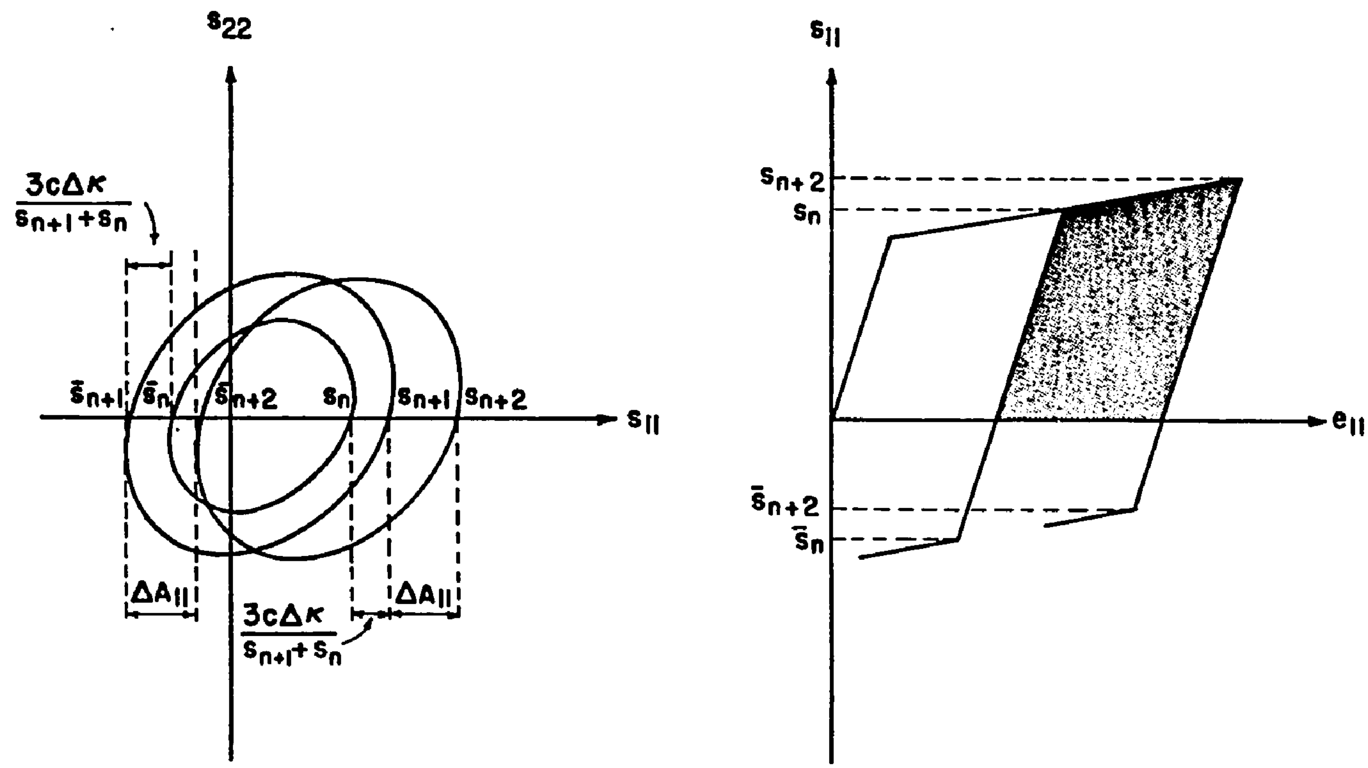

FIGURE 8. CASE OF KINEMATIC AND ISOTROPIC HARDENING

$s_{11(n+2)} - s_{11(n)}$ and $\bar{s}_{11(n+2)} - \bar{s}_{11(n)}$, which represent the applied stress increments, are not equal to ΔA_{11}.

Since the difference of $s_{11(n+1)}$ and $s_{11(n)}$ represents the expansion in the yield surface, it can be obtained from the case of isotropic hardening with the use of some simplifying assumptions. Those assumptions relate directly to equation (77) under the isotropic hardening case. If Green's deformation tensor, C_{AB}, and the determinant of the Jacobian of deformation, J, are assumed to remain unchanged at the beginning and the end of the stress increment, then equation (77) reduces to

$$c = \frac{[C_{11(n)}]^2[J_{(n)}]^{-2}}{3\Delta\kappa} [[s_{11(n+1)}]^2 - [s_{11(n)}]^2] \tag{81}$$

The above assumption is valid when the stress increments are small. Expanding and rearranging equation (81) yields

$$s_{11(n+1)} - s_{11(n)} = \frac{3c\Delta\kappa}{[C_{11(n)}]^2[J_{(n)}]^{-2}[s_{11(n+1)} + s_{11(n)}]} \tag{82}$$

Also, for isotropic hardening we must have

$$\bar{s}_{11(n)} - \bar{s}_{11(n+1)} = \frac{3c\Delta\kappa}{[C_{11(n)}]^2[J_{(n)}]^{-2}[s_{11(n+1)} + s_{11(n)}]} \tag{83}$$

Adding (79) and (82) gives

$$s_{11(n+2)} - s_{11(n)} = \Delta A_{11} + \frac{3c\Delta\kappa}{[C_{11(n)}]^2[J_{(n)}]^{-2}[s_{11(n+1)} + s_{11(n)}]} \tag{84}$$

For small increments of stress $s_{11(n+1)}$ is nearly equal to $s_{11(n)}$. Therefore, equation (84) becomes

$$s_{11(n+2)} - s_{11(n)} = \Delta A_{11} + \frac{3c\Delta\kappa}{2\, s_{11(n)}[C_{11(n)}]^2[J_{(n)}]^{-2}} \tag{85}$$

A similar relationship for the compressive stresses can be obtained by subtracting (83) from (80). This yields

$$\bar{s}_{11(n+2)} - \bar{s}_{11(n)} = \Delta A_{11} - \frac{3c\Delta\kappa}{2\, s_{11(n)}[C_{11(n)}]^2[J_{(n)}]^{-2}} \qquad (86)$$

The unknown quantities in equations (85) and (86) are ΔA_{11} and the parameter c. Adding (85) to (86) gives

$$\Delta A_{11} = \frac{1}{2}\,[s_{11(n+2)} - s_{11(n)} + \bar{s}_{11(n+2)} - \bar{s}_{11(n)}] \qquad (87)$$

The expression for the parameter c can now be obtained by substitution for ΔA_{11} from (87) into either (85) or (86). Solving for c from (87) and (85), we obtain

$$c = \frac{s_{11(n)}[C_{11(n)}]^2[J_{(n)}]^{-2}[s_{11(n+2)} - s_{11(n)} - \bar{s}_{11(n+2)} + \bar{s}_{11(n)}]}{3\Delta\kappa} \qquad (88)$$

The purpose of the formulation of this combined hardening case is also to obtain an expression for the material parameter b. Therefore, as in the case of kinematic hardening, equation (62) can be utilized to fulfill this purpose. Note that for equation (62), $\dot{A}_{11}$ and $\dot{e}''_{11}$ are represented by equations (70) and (71) respectively. However, for this hardening case, ΔA_{11} in (70) is given by the expression obtained in (87). Also, $\Delta e''_{11}$ that appears in (71) can be evaluated in a similar manner to that for the kinematic hardening case. Therefore, $\Delta e''_{11}$ for this hardening case is given by

$$\Delta e''_{11} = \frac{E[e_{11(n+2)} - e_{11(n)}] - [s_{11(n+2)} - s_{11(n)}]}{E} \qquad (89)$$

where E is Young's modulus of elasticity, $e_{11(n)}$ is the Lagrangian strain e_{11} at the end of the stress increment for the combined hardening case. By using equations (70) and (88), and (71) and (89), $\dot{A}_{11}$ and $\dot{e}''_{11}$

can be substituted into equation (62) to obtain the expression for the parameter b. Hence, for the combined hardening case, we have

$$b = \frac{E[s_{11(n+2)} - s_{11(n)} + \bar{s}_{11(n+2)} - \bar{s}_{11(n)}]}{2[E(e_{11(n+2)} - e_{11(n)}) - (s_{11(n+2)} - s_{11(n)})]}$$

$$\cdot \left[\frac{2\,(2\,e_{11} + 1)^2}{2\,(2\,e_{11} + 1)^2 + (2\,e_{22} + 1)^2}\right] \tag{90}$$

where e_{11} and e_{22} are taken to be at the beginning of the stress increment under consideration.

The expressions for the parameters c and b represented by (88) and (90) respectively can be used to evaluate c and b in the case of combined isotropic and kinematic hardening. Equation (88) and (90) are used for several consecutive increments of stresses and strains to obtain several values of c and b. The results for each parameter are then averaged to determine more realistic values of the material parameters c and b.

Determination of Material Response Function

Consider the one-dimensional behavior of equation (20) in terms of the yield function F and assuming $\phi(F) = F$, we obtain

$$\dot{e}''_{11} = \frac{2}{\sqrt{3}}\,\gamma J F\,\sqrt{F + 1}. \tag{91}$$

Equations (6) and (91) state that the material response function $\gamma(F)$ is

$$\gamma(F) = \frac{\sqrt{3}}{2J}\left[\frac{a_1}{(F)^{1-n_1}\,\sqrt{F + 1}} + \frac{a_2(F)^{n_2-1}}{(F_L - F)^{n_3}\,\sqrt{F + 1}}\right] \tag{92}$$

The general character of equation (92) is that γ approaches infinity at the asymptotes $F = 0$ and $F = F_L$ ((see figure 1). This behavior can also be represented by expression (10)

$$\gamma(F) = \frac{\bar{a}_1}{F^{n_1} (F_L - F)^{n_2}} \tag{93}$$

The material function $\gamma(F)$ is determined experimentally for an isothermal state (room temperature). The range of the strain rate used is from 10^{-5} to 10^{-1} per second. The uniaxial yield stress at the corresponding initial yield for the range of the above strain rates is plotted versus the corresponding strain rates along with the a curve fit using equation (93) as shown in Figure 9. The parameters $\bar{a}_1 = 11$, $n_1 = 0.33$, and $n_2 = 0.08$ are obtained using standard statistical procedure (SAS [41], $R^2 = 0.88$).

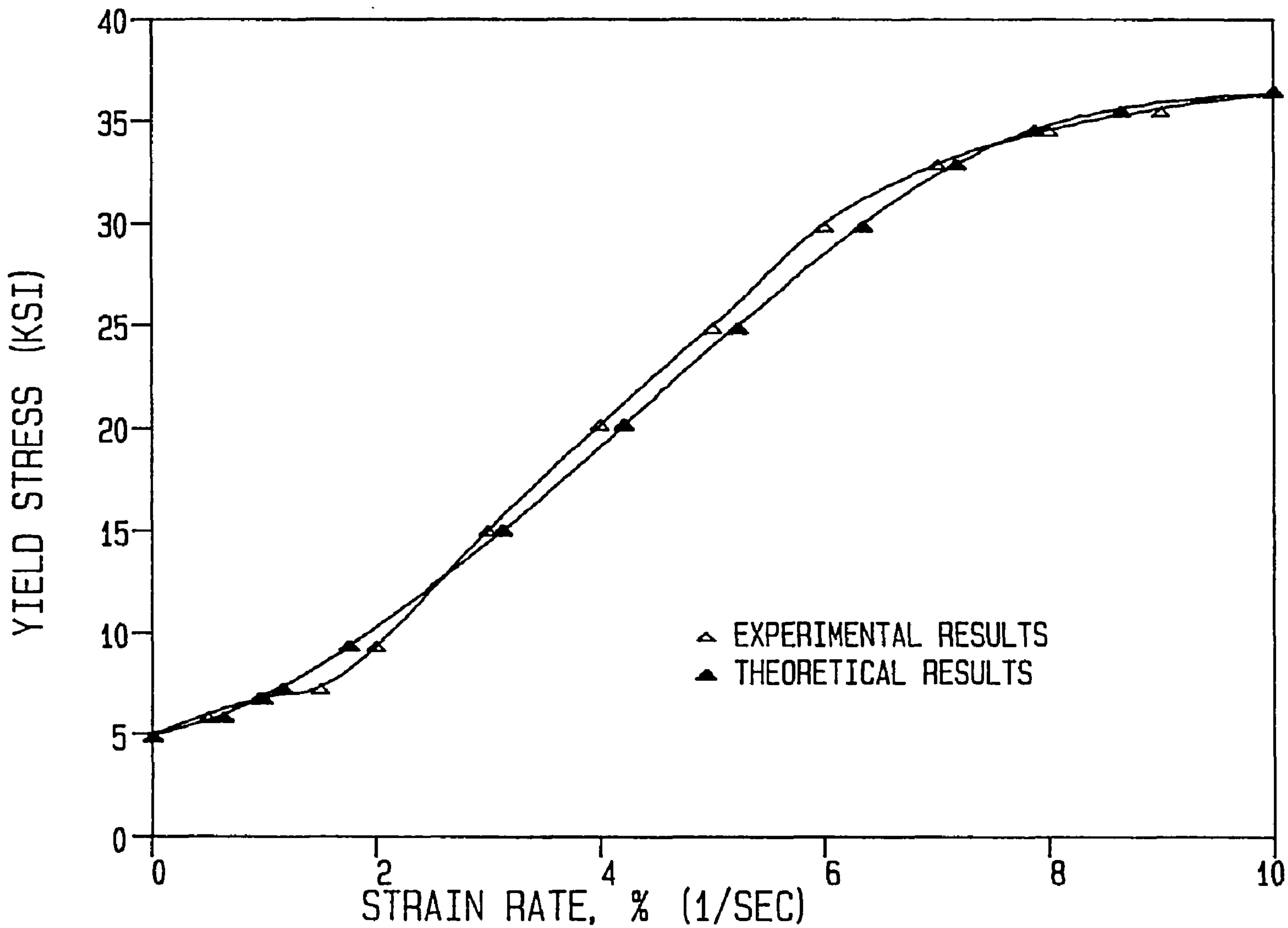

Figure 9. One-Dimensional Yield Stress Versus Strain Rate, Pure Aluminum

Chapter 4

NUMERICAL SOLUTIONS

Uniaxial Test

The theoretical formulations of the constitutive model presented in Chapter 2 is numerically implemented herein for the case of uniaxial loading and reverse-loading. The FORTRAN computer program SAVPA provided in Appendix A is utilized to obtain the numerical solutions. Program SAVPA can yield various numerical solutions of the uniaxial loading and reverse-loading cases. The material parameters b, c, $\gamma(F)$, $\bar{a}_1$, n_1, and n_2, discussed in Chapter 3, are incorporated into the program.

A wide range of numerical solutions may be obtained from the program due to the fact that several choices of the material parameters b and c may be used.

It is important to note that simulation of the experimental data for the uniaxial test requires the use of specific values of parameters b and c for each hardening behaviour assumed. However, for the purpose of studying the response of the constitutive model to various material parameters, several values of b and c can be used for the same hardening case assumed. The study here will be confined to the specific values of the parameters b and c for each hardening case that are required to simulate the experimental data.

A detailed discussion of program SAVPA will not be provided here to avoid repetition. The program is well explained and documented in the Appendix A through the use of comment statements in the program. However, a brief description of the function of each subroutine used in the program, as well as the main program, is provided in Appendix B. Moreover, a discussion regarding the theoretical formulation employed in subroutines YIELDF, PFWRTS, and PEDWTS , which is used in this program, will also be provided in Appendix B. Although the subroutines are employed in the program are used to solve plane-stress problems, i.e., the uniaxial stress problems, they can also be used to solve plane-strain and axi-symmetric problems.

The logical sequence of the computer statements in each program is based on incrementing the material strain that corresponds to a constant spatial strain rate for a particular experiment and solving for the second Piola-Kirchhoff stress. A discussion is provided below on the method of incrementing the material strain for the uniaxial stress problem.

The material strain tensor is related to the displacement field as [38]

$$e_{AB} = \frac{1}{2}\left[\frac{\partial u_A}{\partial x_B} + \frac{\partial u_B}{\partial x_A} + \delta_{CD}\frac{\partial u_C}{\partial x_A}\frac{\partial u_D}{\partial x_B}\right] \tag{94}$$

Therefore, the material strains are represented by:

$$e_{11} = \frac{1}{2}\left[2\left(\frac{\partial u_1}{\partial x_1}\right) + \left(\frac{\partial u_1}{\partial x_1}\right)^2 + \left(\frac{\partial u_2}{\partial x_1}\right)^2 + \left(\frac{\partial u_3}{\partial x_1}\right)^2\right] \tag{95}$$

$$e_{22} = \frac{1}{2}\left[2\left(\frac{\partial u_2}{\partial x_2}\right) + \left(\frac{\partial u_1}{\partial x_2}\right)^2 + \left(\frac{\partial u_2}{\partial x_2}\right)^2 + \left(\frac{\partial u_3}{\partial x_2}\right)^2\right] \tag{96}$$

$$e_{33} = \frac{1}{2}\left[2\left(\frac{\partial u_3}{\partial x_3}\right) + \left(\frac{\partial u_1}{\partial x_3}\right)^2 + \left(\frac{\partial u_2}{\partial x_3}\right)^2 + \left(\frac{\partial u_3}{\partial x_3}\right)^2\right] \tag{97}$$

$$e_{12} = \frac{1}{2}\left[\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_1}\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_1}\frac{\partial u_3}{\partial x_2}\right] \tag{98}$$

$$e_{23} = \frac{1}{2}\left[\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2} + \frac{\partial u_1}{\partial x_2}\frac{\partial u_1}{\partial x_3} + \frac{\partial u_2}{\partial x_2}\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\frac{\partial u_3}{\partial x_3}\right] \tag{99}$$

$$e_{31} = \frac{1}{2}\left[\frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3} + \frac{\partial u_1}{\partial x_1}\frac{\partial u_1}{\partial x_3} + \frac{\partial u_2}{\partial x_1}\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\frac{\partial u_3}{\partial x_3}\right] \tag{100}$$

The complete displacement field in a uniaxial state of stress problem is not defined. Consequently, the material strain tensor cannot be obtained directly from equations (94) through (100). The relationship between the material strain e_{11} and, e_{22} and e_{33}, is not directly attainable. However, the relationship between the elastic components of the material strain increments, and the relationship between the plastic components of the material strain increments are defined.

The decomposition of the material strain increments Δe_{11}, Δe_{22}, and Δe_{33} into their elastic and viscoplastic components is represented as follows:

$$\Delta e_{11} = \Delta e'_{11} + \Delta e''_{11} \tag{101}$$

$$\Delta e_{22} = \Delta e'_{22} + \Delta e''_{22} \tag{102}$$

$$\Delta e_{33} = \Delta e'_{33} + \Delta e''_{33} \tag{103}$$

From the axi-symmetric nature of the uniaxial test, Δe_{22}, $\Delta e'_{22}$, and $\Delta e''_{22}$ are equal to Δe_{33}, $\Delta e'_{33}$, and $\Delta e''_{33}$ respectively. To obtain the relationship between the elastic strain increments $\Delta e'_{11}$ and $\Delta e'_{22}$, consider the following general elastic relationship between the stress increment Δs_{AB} and the elastic strain increments $\Delta e'_{CD}$

$$\Delta s_{AB} = E_{ABCD}\, \Delta e'_{CD} \tag{104}$$

where E_{ABCD} is the elastic stiffness tensor defined in equation (37) in Chapter 2. It follows from equation (104) for a uniaxial state of stress, that

$$\Delta s_{22} = E_{2211}\, \Delta e'_{11} + E_{2222}\, \Delta e'_{22} + E_{2233}\, \Delta e'_{33} = 0 \tag{105}$$

Since $\Delta e'_{22}$ is equal to $\Delta e'_{33}$ as discussed above, we can solve for $\Delta e'_{22}$ in terms of $\Delta e'_{11}$ from equation (105). Therefore, we obtain the useful relationship

$$\Delta e'_{22} = -\left(\frac{E_{2211}}{E_{2222} + E_{2233}}\right) \Delta e'_{11} \tag{106}$$

Equation (106) can be expressed in terms of Lame's constant λ and G using equation (37), as follows:

$$\Delta e'_{22} = -\frac{\lambda}{2(\lambda + G)} \Delta e'_{11} \tag{107}$$

The relationship between the viscoplastic components of the material strain increments $\Delta e''_{11}$ and $\Delta e''_{22}$ follows from equations (58), (60), and (61) in Chapter 3. That is, due to the absence of viscoplastic volumetric strains, we have

$$\Delta e''_{22} = -\frac{2e_{22} + 1}{2(2e_{11} + 1)} \Delta e''_{11} \tag{108}$$

where e_{11} and e_{22} are the material strains before applying the strain increments Δe_{11}, Δe_{22}, and Δe_{33}.

The procedure of incrementing the material strains in program SAVPA is outlined here to help the user follow the sequence of the computer statements. The material strain increment Δe_{11}, that corresponds to a constant spatial strain rate for a given experiment is calculated. If the material is undergoing elastic deformation, then Δe_{22} is obtained directly from equation (106) by replacing $\Delta e'_{11}$ and $\Delta e'_{22}$ with Δe_{11} and Δe_{22} respectively. However, when the material is in a state of viscoplastic flow, the elastic and viscoplastic components of Δe_{11} need to be determined first. $\Delta e''_{11}$ can be determined from equation (20) in Chapter 2. In terms of the $\Delta e''_{11}$ increment, equation (20) is written as

$$\Delta e''_{11} = J\gamma F \frac{\partial F}{\partial s_{11}} \tag{109}$$

where the right hand side of equation (109) is computed by the program based on the state of stress and strain of the material just before applying the strain increment Δe_{11}. Now, $\Delta e'_{11}$ is easily obtained from equation (101) and (109). That is, we have

$$\Delta e'_{11} = \Delta e_{11} - J\gamma F \frac{\partial F}{\partial s_{11}} \tag{110}$$

Once $\Delta e'_{11}$ and $\Delta e''_{11}$ are determined, $\Delta e'_{22}$ and $\Delta e''_{22}$ can be obtained from equations (107) and (106) respectively. Hence, Δe_{22}, which is equal to Δe_{33}, is obtained from equation (102). Now that Δe_{11}, Δe_{22}, and Δe_{33} are all known, the constitutive model derived in Chapter 2, equations (44) and (45), can be used to determine the increment of stress s_{11}, and subsequently, the total stress s_{11}.

Bending of a Moderately Thick Plate

The applicability and effectiveness of the proposed constitutive model in solving complex finite deformation problems is demonstrated by the numerical example of bending of a moderately thick plate. The problem of analysis of displacements, stresses, and strains in elements made of commercially pure aluminum subject to arbitrarily large deformations under the condition of plane strain is formulated in terms of the finite element method.

From equation (45), the elasto/viscoplastic tangent modulus is expressed as

$$D_{RSPQ}{}^{(n)} = N_{RSAB}{}^{(n)} \, E_{ABPQ}. \tag{111}$$

In order to obtain the tangent visco-plastic stiffness matrix, use is made of the equilibrium equation at any instant of time $t^{(n)}$, where

$$\int_{V_0} [\underset{\sim}{B}^{(n)}]^T \, \underset{\sim}{s}^{(n)} \, dV - \underset{\sim}{R}^{(n)} = 0 \tag{112}$$

where $\underset{\sim}{R}^{(n)}$ is the nodal force vector due to body forces and surface tractions, and $\underset{\sim}{B}^{(n)}$ is the kinematic large displacement matrix relating the increments of total strain to displacement increments. Equation (112) in incremental form is expressed as [42,43,44]

$$\int_{V_0} [\underset{\sim}{B}^{(n)}]^T \Delta\underset{\sim}{s}^{(n)} \, dV_0 + \underset{\sim}{K}_{\sigma}^{(n)} \Delta\underset{\sim}{u}^{(n)} - \Delta\underset{\sim}{R}^{(n)} = 0 \qquad (113)$$

where $\underset{\sim}{K}_{\sigma}^{(n)}$ is the initial stress, or geometric stiffness matrix [44] dependent on the stress level and $\Delta\underset{\sim}{R}^{(n)}$ is the change in the external loads during the time increment $\Delta t^{(n)}$. The matrix $\underset{\sim}{B}^{(n)}$ may be expressed as

$$\underset{\sim}{B}^{(n)} = \underset{\sim}{B}_L^{(n)} + \underset{\sim}{B}_{NL}^{(n)} \qquad (114)$$

where $\underset{\sim}{B}_L^{(n)}$ and $\underset{\sim}{B}_{NL}^{(n)}$ are the linear and nonlinear terms, respectively, of the general quadratic relationship between strains and displacements in the material formulation.

Substituting from equation (41) of Chapter 2 into equation (113), we obtain

$$[\hat{\underset{\sim}{K}}^{(n)} + \underset{\sim}{K}_{\sigma}^{(n)}] \Delta\underset{\sim}{u}^{(n)} = \Delta\underset{\sim}{R}^{(n)} + \int_{V_0} [\underset{\sim}{B}^{(n)}]^T \underset{\sim}{D}^{(n)} \dot{\underset{\sim}{e}}'' \Delta t^{(n)} \, dV_0 \qquad (115)$$

or

$$\underset{\sim}{K}^{(n)} \Delta\underset{\sim}{u}^{(n)} = \Delta\bar{\underset{\sim}{R}}^{(n)} \qquad (116)$$

where

$$\underset{\sim}{K}^{(n)} = \hat{\underset{\sim}{K}}^{(n)} + \underset{\sim}{K}_{\sigma}^{(n)} \qquad (117)$$

$$\hat{\underset{\sim}{K}}^{(n)} = \int_{V_0} [\underset{\sim}{B}^{(n)}]^T \underset{\sim}{D}^{(n)} \underset{\sim}{B}^{(n)} \, dV_0 \qquad (118)$$

$$\Delta\bar{\underset{\sim}{R}}^{(n)} = \Delta\underset{\sim}{R}^{(n)} + \int_{V_0} [\underset{\sim}{B}^{(n)}]^T \underset{\sim}{D}^{(n)} \dot{\underset{\sim}{e}}'' \Delta t^{(n)} \, dV_0 \qquad (119)$$

From equation (116) we can solve for $\Delta\underset{\sim}{u}^{(n)}$ and determine $\underset{\sim}{u}^{(n+1)}$, $\underset{\sim}{s}^{(n+1)}$, $e''^{(n+1)}$ and $e^{(n+1)}$, where

$$\begin{aligned}
\underset{\sim}{u}^{(n+1)} &= \underset{\sim}{u}^{(n)} + \Delta \underset{\sim}{u}^{(n)} \\
\underset{\sim}{s}^{(n+1)} &= \underset{\sim}{s}^{(n)} + \Delta \underset{\sim}{s}^{(n)} \\
\underset{\sim}{e}''^{(n+1)} &= \underset{\sim}{e}''^{(n)} + \Delta \underset{\sim}{e}''^{(n)} \\
\underset{\sim}{e}^{(n+1)} &= \underset{\sim}{e}^{(n)} + \Delta \underset{\sim}{e}^{(n)}
\end{aligned} \tag{120}$$

An equilibrium correction is obtained because the stresses obtained by adding the stress increments that are calculated from equation (113) are not strictly correct [39] and will not satisfy the equilibrium equation (112). The following approach is used for applying the necessary correction [39,45]. Using the following expression we compute $\Delta \underset{\sim}{s}^{(n)}$,

$$\Delta \underset{\sim}{s}^{(n)} = \underset{\sim}{D}^{(n)} \left(\underset{\sim}{B}^{(n)} \Delta \underset{\sim}{u}^{(n)} - \dot{\underset{\sim}{e}}''^{(n)} \Delta t^{(n)}\right) \tag{121}$$

where $\Delta \underset{\sim}{u}^{(n)}$ is obtained from equation (116). Substituting for $\Delta \underset{\sim}{s}^{(n)}$ in equation (120), we obtain $\underset{\sim}{s}^{(n+1)}$. The stress $\underset{\sim}{s}^{(n+1)}$ substituted in equation (112) at time $t^{(n+1)}$ results in the residual load $\underset{\sim}{F}^{(n+1)}$ where

$$\underset{\sim}{F}^{(n+1)} = \underset{\sim}{R}^{(n+1)} - \int_{V_0} [\underset{\sim}{B}^{(n+1)}]^T \underset{\sim}{s}^{(n+1)} \, dV \tag{122}$$

This residual load may either be added to the next force increment at the next time step or used in an iterative process in order to obtain a reduction in error accumulation.

The above formulation is included in an existing finite element program [35].

The plate dimensions and the finite element mesh are shown in Figure 10. The experimental set up of the plate is shown in Figure 11. The plate is loaded by a line force at the mid-span of the simply supported plate. The deflection at the bottom center of the plate is measured using linear variable differential transformer (LVDT). The plate supports are kept at constant locations and the effect of the plate sliding on the supports have been included in the numerical procedure.

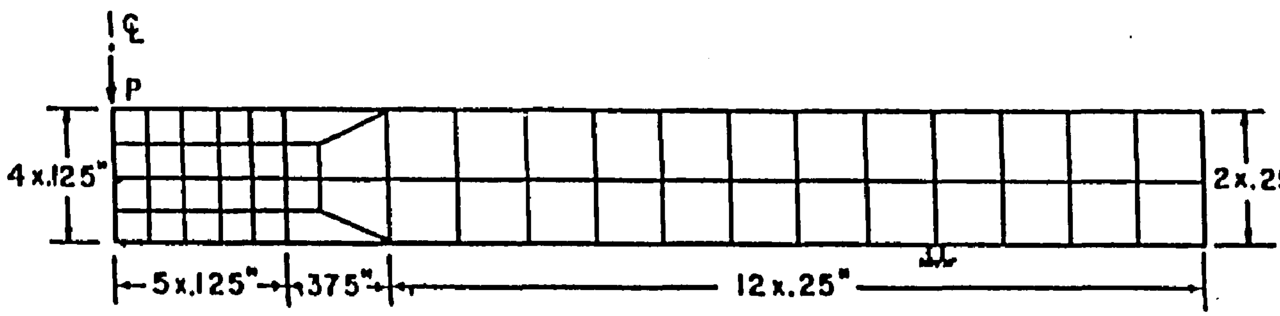

Figure 10. Plate Dimension and Finite Element mesh

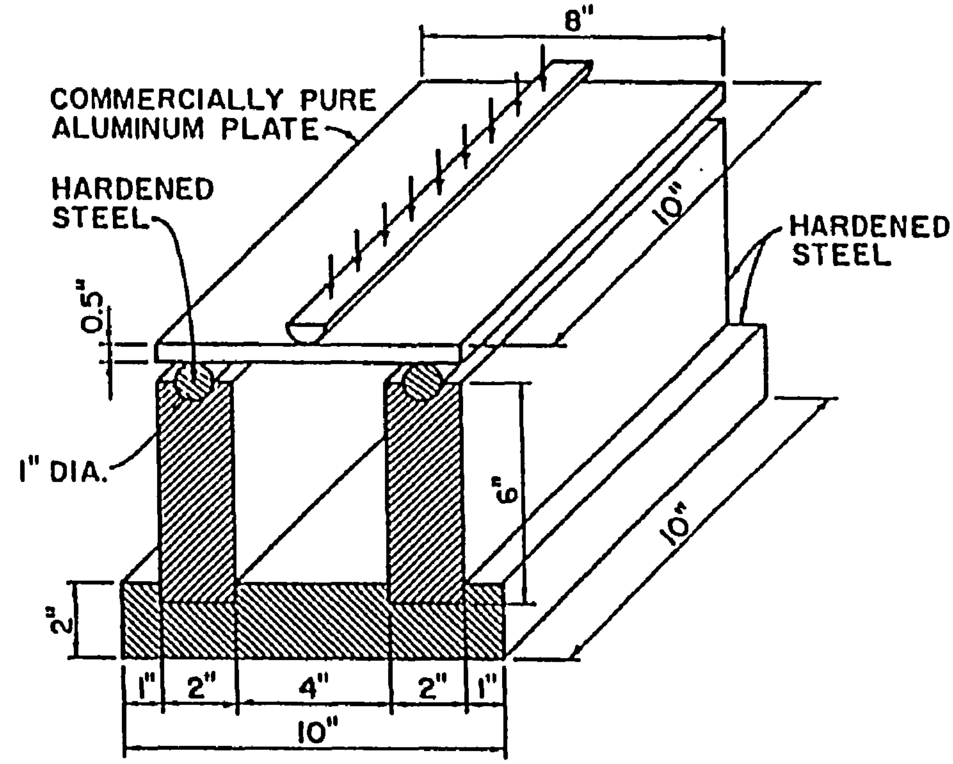

Figure 11. EXPEREMENTAL SET-UP FOR THE PLATE

Chapter 5

DATA ACQUISITION AND EQUIPMENT CONTROL

The recent development of microcomputers together with the development of interfacing boards between digital and analog equipment now provides a relatively inexpensive way to build sophisticated apparatus where tasks like waveform generation, data acquisition, equipment control and data processing can be accomplished almost simultaneously.

Hardware

To successfully perform all the required tasks, a microcomputer (IBM compatible XT) is used together with an interfacing board from Metrabyte and a signal conditioning interface from MTS Corporation. The diagram in Figure 12 shows schematically how several parts of the testing apparatus are interconnected.

IBM Compatible PC/XT. The microcomputer used is an IBM Compatible (Zenith Z-148) PC/XT that is based on a high performance 16 bit Intel 8086 microprocessor equipped with low density 360kb diskette drive, a 20Mb fixed disk drive, 640 Kb of RAM, and a math co-processor 8087. This model contains option slots that support features cards for additional devices.

An enhanced color graphics display provides an enhanced level color graphics with 640 x 350 pel definition. A graphics printer provides hard copies of the graphics and the text.

DASH16 Metrabyte Board. Metrabyte's DASH16 is a multi-function high speed analog/digital/ input/output expansion board for an IBM PC. It is a full length board that installs internally in an expansion slot

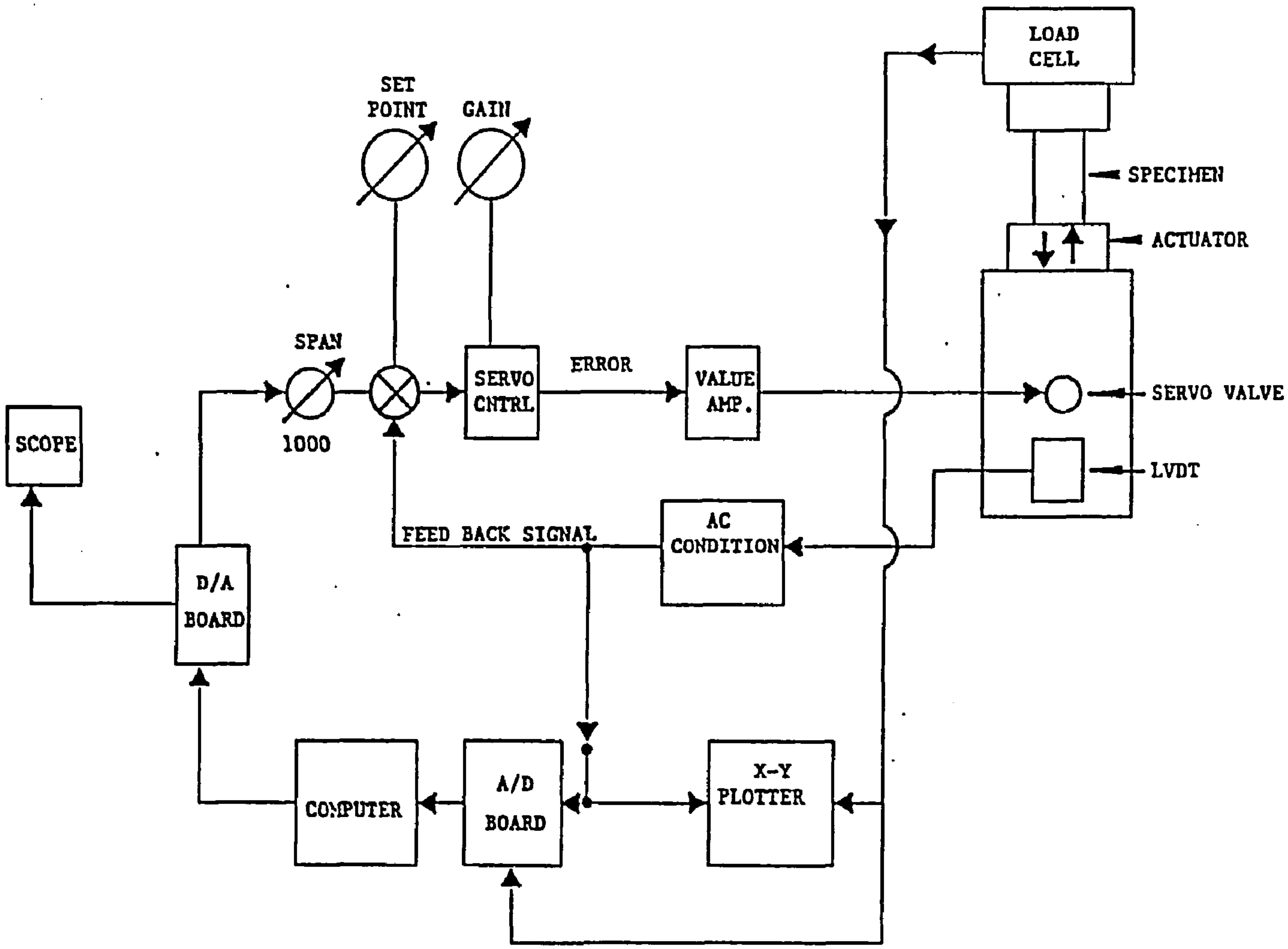

Figure 12. Dynamic Loading System and Schematic of Data Acquisition and Control

of an IBM compatible PC/XT and which transforms the computer into a fast, high precision data acquisition and signal analysis instrument.

DASH16 uses an industry standard (AD674A) 12 bit successive approximation converter with a 25 microsecond conversion time. The channel input configuration is switch selectable on the board providing a choice between 16 single ended channels or 8 differential channels. Throughput using DMA (Direct Memory Access) channels can be up to 50,000 conversions/second. Direct memory access is the only satisfactory way of transferring data from the A/D converter to the computer memory at rates above 10,000 samples/second. Real time triggering of A/D assures perfect synchronization in sampling, unaffected by other computer operations. These capabilities are essential for equipment control.

DASH16 has two channels of multiplying 12 bit D/A output. These converters are double buffered to provide instantaneous update.

Signal Conditioning Interface. To interface servovalve, strain gauges, and LVDTs to the Metrabyte board it was necessary to use specialized electronic equipment. Essentially this equipment is divided into two parts:

(a) One part is an MTS signal conditioning unit (406 controller and 408 DC conditioner) that provides the excitation for the load cell, stroke transducer, and extensometer. It amplifies the dc error signal (the difference between the composite command signal and the transducer conditioner output (feedback)) which becomes the servovalve control signal.

(b) The second part involves the construction of a transducer signal conditioner. It provided eight channels of amplification of the transducer output signals and also supplies an AC excitation voltage to each transducer. All channels were configured for a low level (strain gauge) transducers.

Software

To specifically address all hardware components of the equipment and to allow them to perform all the tasks that the experiment requires, several software programs have been made. The software is written in compiled BASIC version 1.0. However, when higher speeds are required ASSEMBLER subroutine calls are made from inside the BASIC program addressing specific components and/or requesting specific tasks to be performed. As a result, the flexibility of BASIC is enhanced by the speed of ASSEMBLER.

Most of the interface with the Metrabyte DASH16 board was accomplished using the software provided by the manufacturer. Fortunately, Metrabyte provides the source listings of their software, making it relatively easy to change and insert source code so that specialized functions can be performed. The standard driver subroutines remain unmodified, allowing continued compatibility with the programming manual.

To have a clear understanding of the purpose of each of the different software programs, it is necessary to outline the tasks that each program is responsible. While data is being collected, the applied stroke magnitude must be such that a constant spatial strain rate is maintained during the loading and reverse-loading tests. For higher accuracy in test results, feedback closed control loops and high signal resolution must be used. To allow several types of test condition (i.e., loading and reverse-loading at different strain rate, short term creep and relaxation) to be performed sequentially on a single specimen, special software programs were developed to allow for the rapid change of the signal of the feedback channel, since the hydraulic servo arms must always be kept under feedback closed-loop control. Consequently, this forces the adoption of software-programmable gains and software programmable analog voltage offsets.

Furthermore, one should realize that feedback closed loop control derives its accuracy from the fact that the desired signal (COMMAND) is compared with the signal obtained at the transducer conditioner output (FEEDBACK), and a dc error signal is sent to the servovalve so it may make the necessary adjustment.

When analog equipment was used in feedback closed loop controls, the time between reading the feedback signal, comparing it to a command value, sending out the error value and rereading the feedback value again was almost zero, i.e., the rate (number of closed loops/time) was almost infinite. However, using digital equipment, this is not the case due to the time involved in:

(1) converting the feedback signal from analog to digital;

(2) computing the command value;

(3) determining the error signal; and

(4) converting it from digital to analog.

The rate is, therefore, a finite number that has to be kept sufficiently high in order to provide complete control of the actuator.

The menu driven programs, SETUP, BARTST, and PLTTST were developed so that the above requirements could be fulfilled.

BARTST - See Appendix C. This program is used for uniaxial loading reverse-loading tests. It containts proper conversion factors, amplification gains, and analog hardware offsets for each of the input channels in order to convert input voltage from the probes into engineering units. This program permits the execution of tests where the spatial strain and/or load can be linearly varied as a function of time. The program prompts the user with the name of the configuration file (see SETUP) to be used. Any channel can be used as a feedback for the closed loop control. Stress or strain closed loop control is made possible simply by choosing the correct feedback channel. If the feedback is the output of the load cell, then the testing would be under stress control. If the feedback is the output of an LVDT or extensometer, then the testing would be done under strain control.

This program is menu driven with several screens appearing during the test program. The first screen contains basic information which instructs the program with information such as where to store the data, strain rate for loading as well as reverse loading, and specimen dimensions which are needed for the control of the actuator during spatial strain control and for post processing of the data. The second screen allows the user, if desired, to manually zero the output of each transducer.

For BARTST to perform satisfactorily, two ASSEMBLER language MODES, MODE 3 and MODE 15, had to be developed. All testing is actually controlled by MODE 3. Calls using this mode are placed between lines of BASIC code. This solution, although not sophisticated, is quite efficient because the speed of execution in compiled BASIC is sufficiently high to keep the actuator in position.

Every 30 microsecond, DASH16 updates 2 computer memory bytes for each input channel with a digital value corresponding to the voltage present in each input channel. At the end of each second, one representative value of each input is placed in RAM. MODE 3 compares the value present in the bytes, representative of the feedback channels, with the corresponding values in the command wave. Control of the actuator is maintained by sending the difference between the feedback values and the command values, multiplied by a constant (GAIN) at the transducer conditioner of the feedback. Gain depends on the stiffness of the specimen, the oil pressure, the stiffness of the frame, and the frequency and amplitude of the loads. The setting must be determined experimentally in each situation.

BARTST has the capability to perform loading, reverse-loading, short term creep and relaxation test. The loading reverse-loading is achieved by pushing the directional keys on the keyboard, therefore, command values for the actuator may be altered, forcing a change in its position. The hold required for creep and relaxation is done by pressing the HOME key. Pressing any other key will cause the test to resume.

After the completion of each individual test, data is retrieved from RAM and stored on the had disk (for each test 64kb of data can be collected). This operation is accomplished while simultaneously maintaining control of the actuator. This program also utilizes the definition of data acquisition schedules. To accurately perform closed loop control, very high rates of data acquisition are necessary. However, for data analysis only a small number of conversions are required. Saving all the data points would not only be next to impossible, but inefficient. The data acquisition schedule instructs the computer which periods to save for later analyses.

Post-processing programs are used for curve fitting, data analysis and plotting routines.

PLTTST - See Appendix D. This program is used for testing of moderately thick plates. Features in this program are similar to the ones described in BARTST. This program uses a load control type of test only.

Several smaller programs were developed to calibrate and display the calibration curves for transducers.

Chapter 6

DISCUSSION OF RESULTS

Uniaxial Test

Figures 13 through 21 display the numerical and experimental solutions of the uniaxial loading reverse-loading tests for commercially pure aluminum specimens. The uniaxial tension loading tests are conducted for five different constant spatial strain rate, d_{11} (1.8 x 10^{-5}/sec through 10^{-1}/sec). In compression region, only two different constant spatial strain rates are used (10^{-4}/sec and 10^{-2}/sec). In Figures 13 through 17, the second Piola-Kirchhoff stress s_{11} versus the Lagrangian strain e_{11} are plotted, whereas in Figures 18 through 22, the Cauchy stress t_{11} versus the Lagrangian strain e_{11} are plotted. Based on the comparison of the numerical results with the experimental tests, we find that the case of combined isotropic and kinematic hardening best simulate the true behavior of the commercially pure aluminum.

It is noted in Figures 13 through 17 that the slope of the elastic unloading line is less steep than the slope of the initial elastic loading line. This effect increases with plastic flow. In Figures 18 through 22, the magnitude of the slope of the elastic unloading lines is close to the magnitude of the slope of the initial elastic loading line. This primarily true because Cauchy stress takes into account the change in the cross- sectional area of the specimen. In the case of the second Piola- Kirchhoff stress, this phenomenon has to be reflected through the elastic stiffness tensor since the definition of the stress tensor itself does not incorporate the change in the cross sectional area. In order to remedy this phenomenon, Voyiadjis [46] has suggested a damage factor to be incorporated in the elastic stiffness. Nevertheless, in the present work, the elastic stiffness is assumed constant.

In Figures 13 through 22, the magnitude of the rate of hardening decreases as the strain rate increases at large strains. In these figures a constant spatial strain rate k_1 is maintained for every uniaxial test during loading or unloading. Therefore, we have

$$d_{11} = k_1 \tag{123}$$

where,

$$d_{11} = \frac{\dot{\varepsilon}_{11}}{1 + \dot{\varepsilon}_{11}} \tag{124}$$

and hence,

$$\dot{\varepsilon}_{11} = k_1 \ (1 + \varepsilon_{11}) \tag{125}$$

where ε_{11} is the engineering strain.

Bending of Moderately Thick Plate

The load-deflection curves obtained from the finite element analysis and the experimental load-deflection curves are shown in Figure 23. Two different loading rates of 6.1 and 122.1 lb/sec are used for the bending of the thick plate. The one-half inch thick plates are deflected up to a maximum of 0.87 inches deflection at the bottom center of the plate.

It should be noted that the computation is based on the assumption of plane strain while in the tested plate the state of stress and strain is three-dimensional. In Figure 23 we observe the experimental curve as expected is below the computed curve. Strains of the order of 25% were exhibited in the plate bending problem.

Creep and Relaxation

The creep and relaxation behavior of the commercially pure aluminum is also investigated. For the case of the uniaxial loading specimens, at a constant spatial strain rate of $d_{11} = 10^{-1}$ per second, a relaxation period of one hour is used. In Figure 24 we note that during this period of relaxation time, the stress dropped by 5.00 psi from a stress level of 23.00 psi to 18.00 psi. The corresponding total axial material strain during this drop in stress level is 1.54 percent.

The creep behavior of this material is shown in Figure 25 for the case of uniaxial loading. The creep strain is investigated at a uniaxial stress level of 22,113 psi. In Figure 25 we note that for a time period of one hour, the material creep strain attained a value of 0.36 percent.

The present constitutive formulation does not provide the capability of computing the creep strain. Expressions presented by Liu and Krempl [25], Kujawski, Kallianpur, and Krempl [47], and Krempl [48] may be used to compute the creep strain.

The creep effect is also investigated for the case of bending of the thick plate of thickness 0.50 inch shown in Figure 10. In Figure 26 the creep effect is demonstrated for a plate subjected to a line load that is loaded at a rate of 30.0 lb/sec. After maintaining a constant load of 4800.0 lb for a period of 67 minutes, the creep deflection is measured to be 0.05 inch. The creep deflection versus time is shown in Figure 27 for the bending of the thick plate.

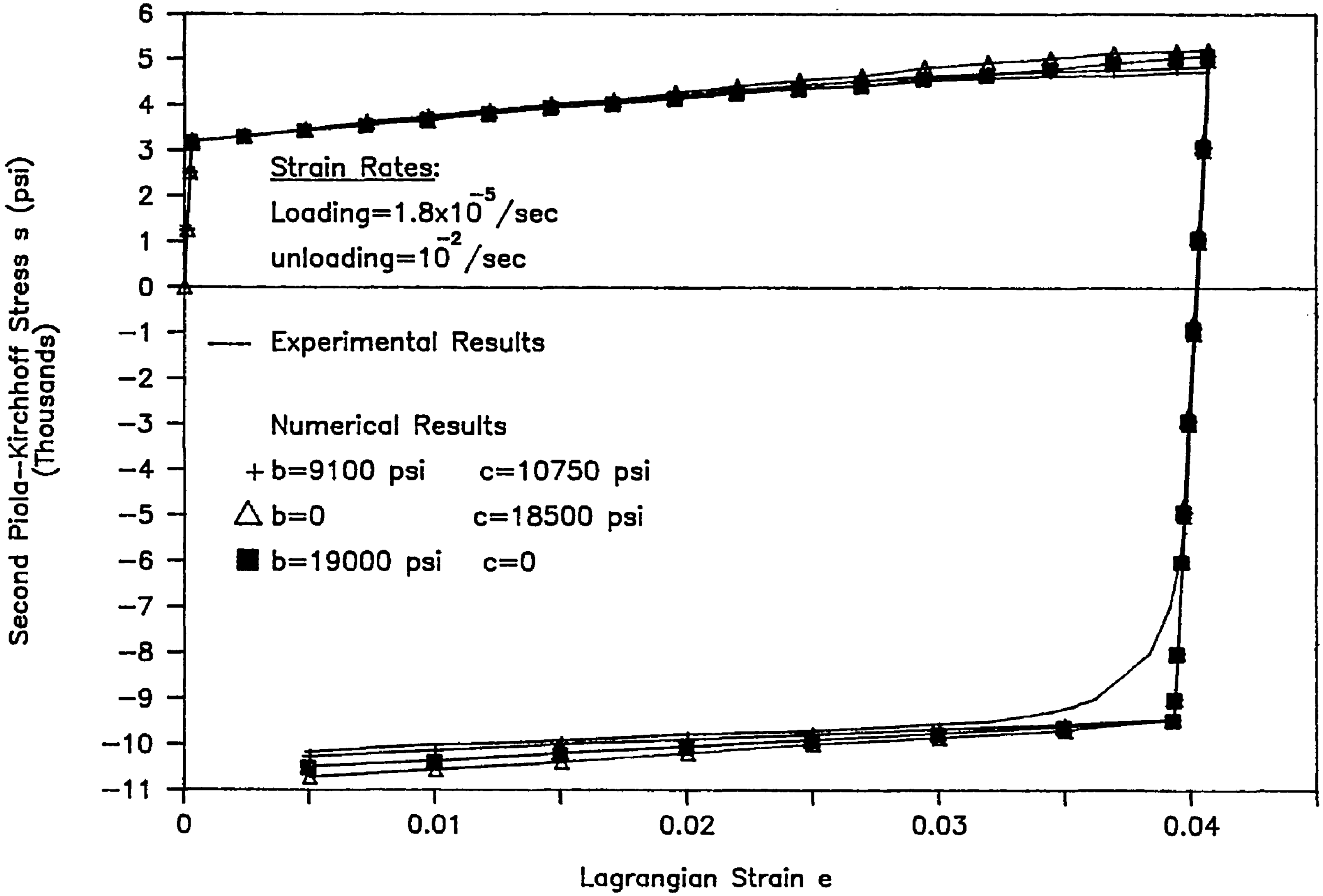

Figure 13. Experimental & Theoretical Relations Between e_{11} & s_{11}

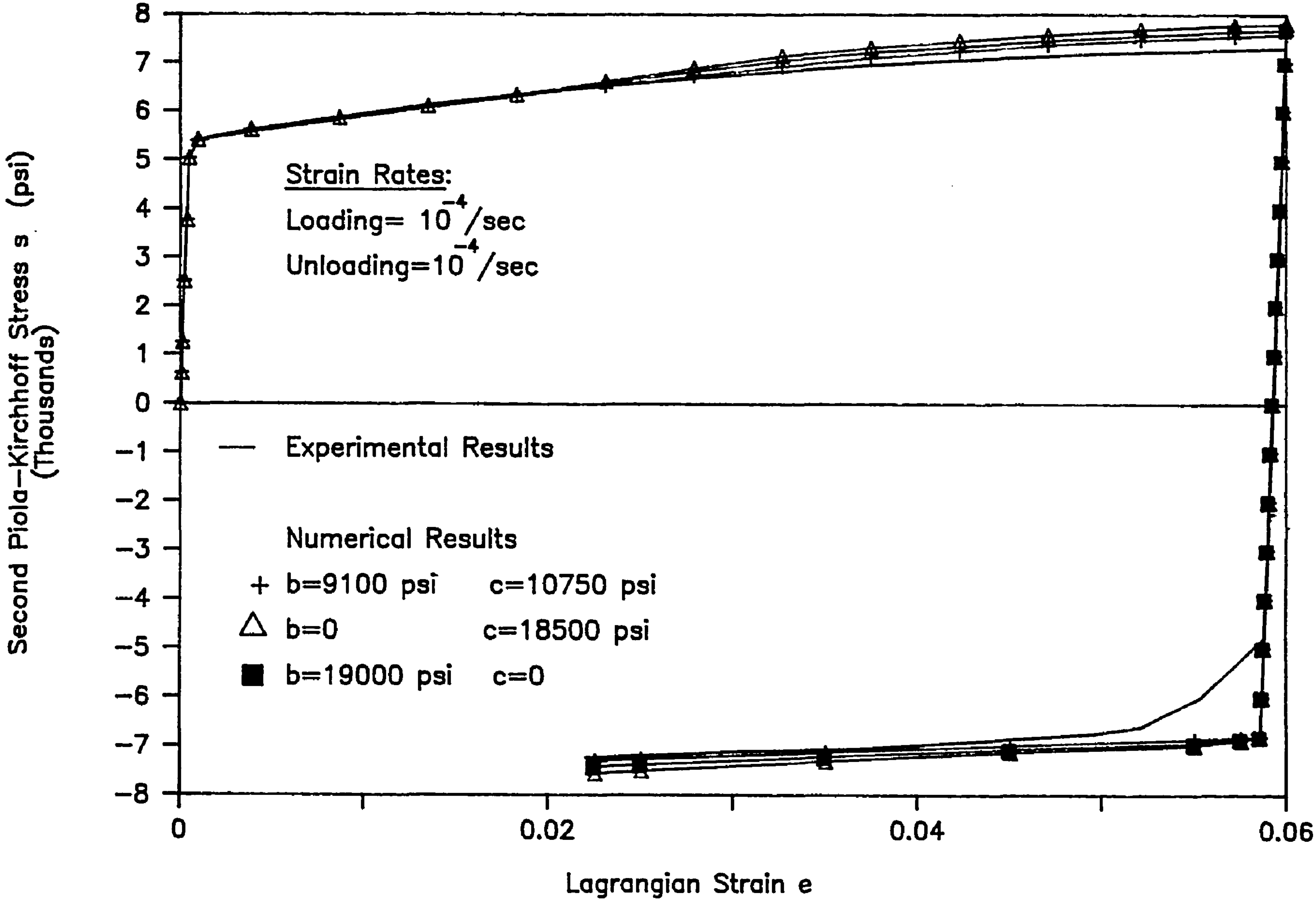

Figure 14. Experimental & Theoretical Relations between e_{11} & s_{11}

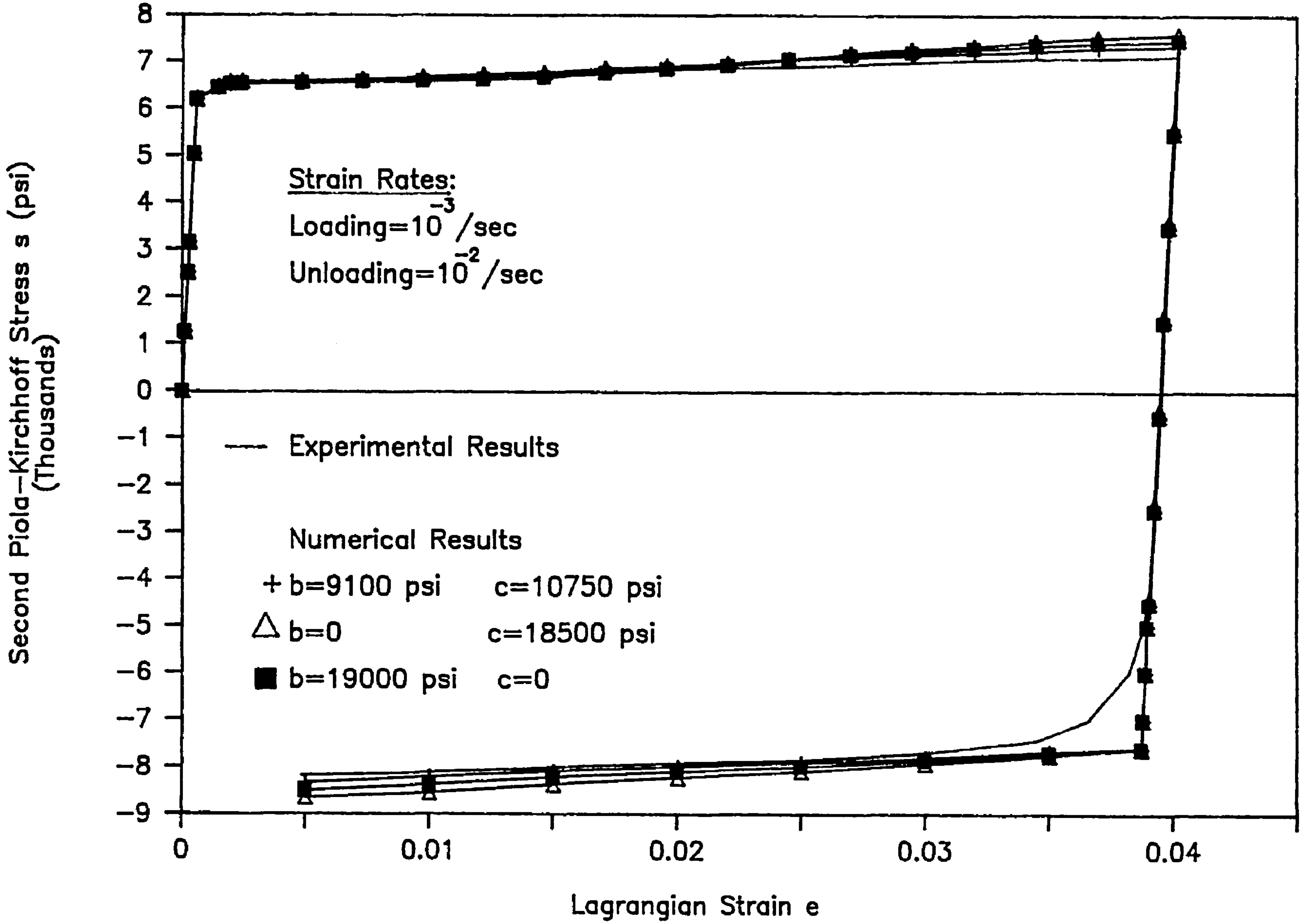

Figure 15. Experimental & Theoretical Relations Between e_1 & s_1

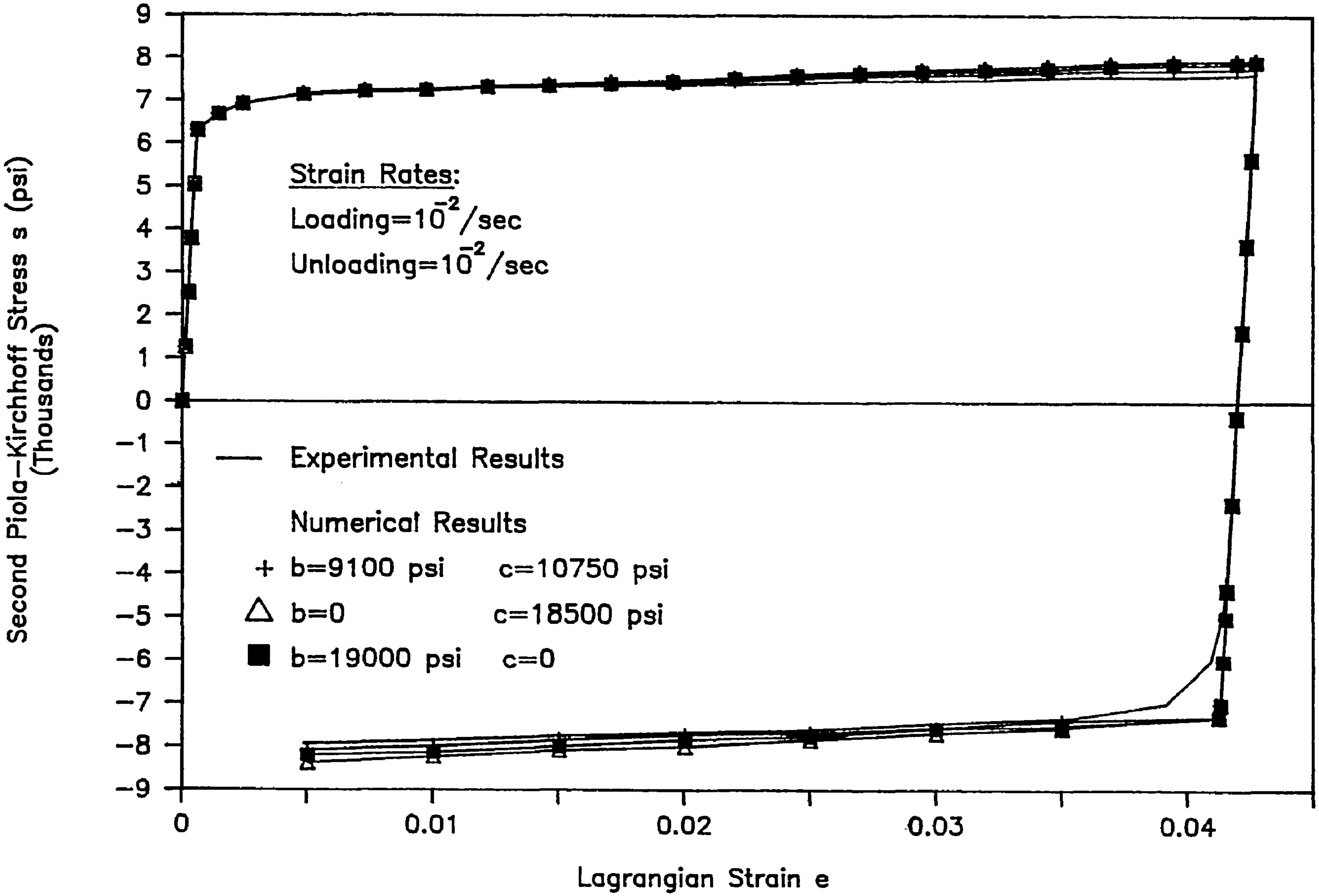

Figure 16. Experimental & Theoretical Relations Between e_{11} & s_{11}

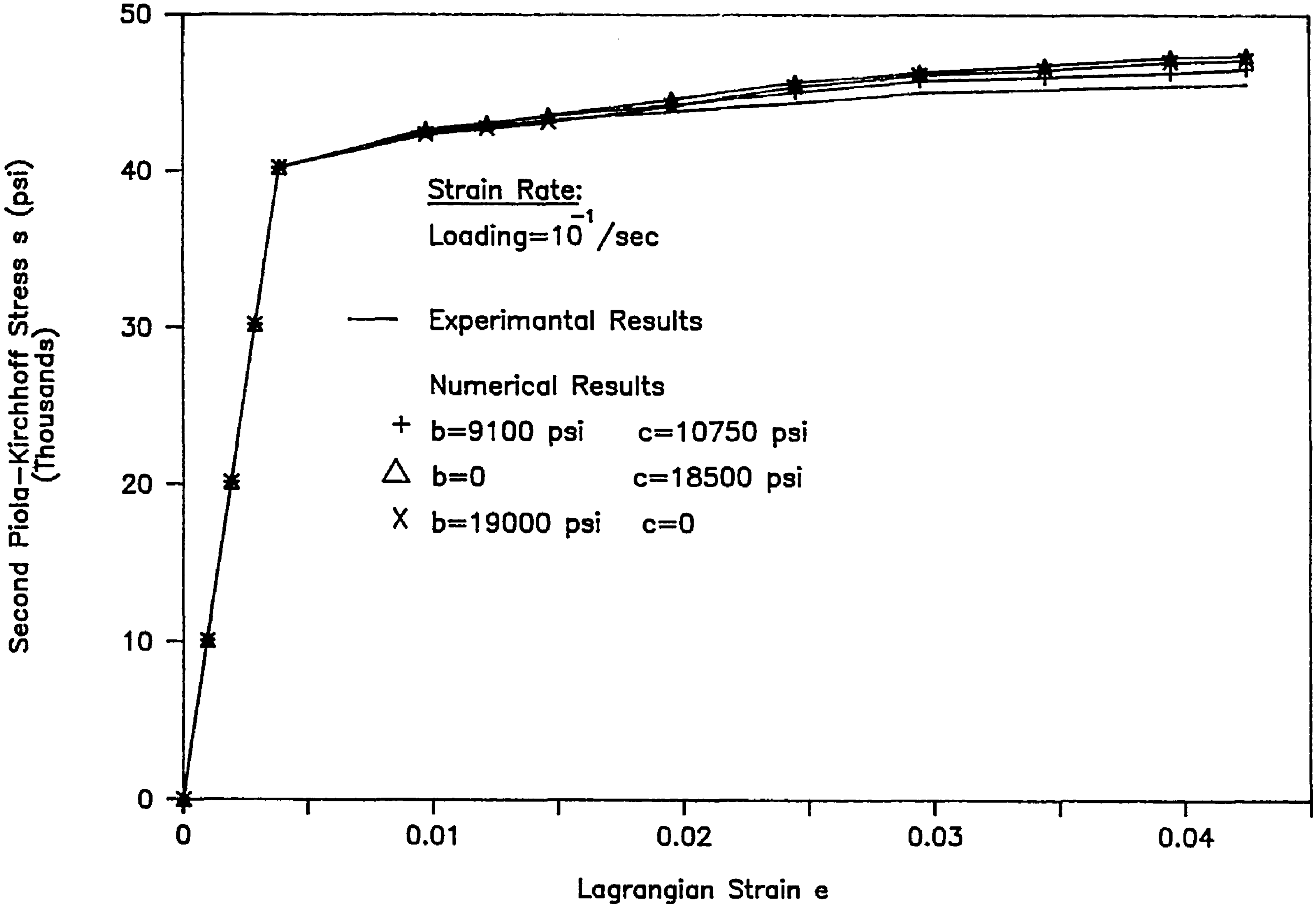

Figure 17. Experimental & Theoretical Relations Between e_{11} & s_{11}

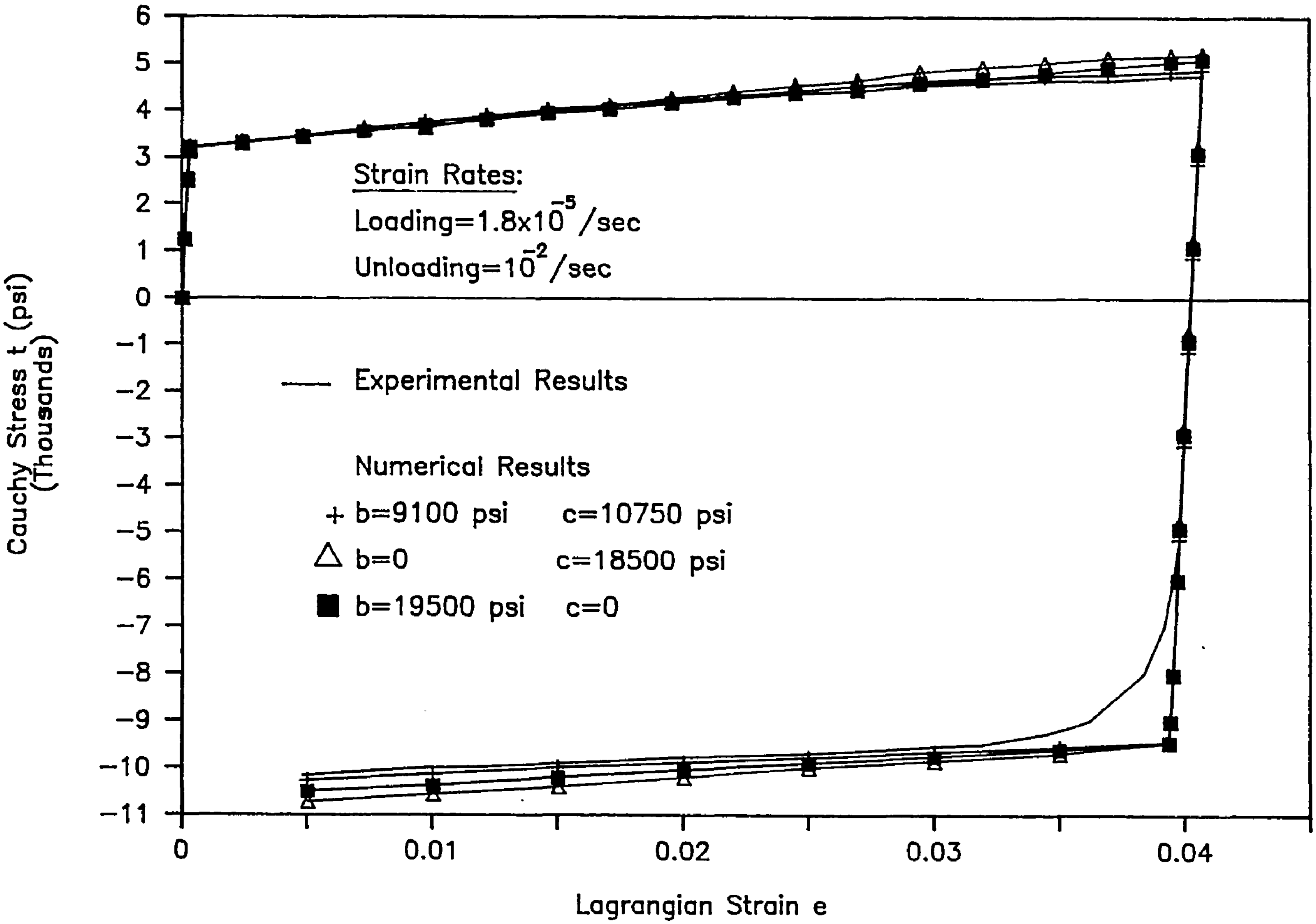

Figure 18. Experimental & Theoretical Relations Between e_{11} & t_{11}

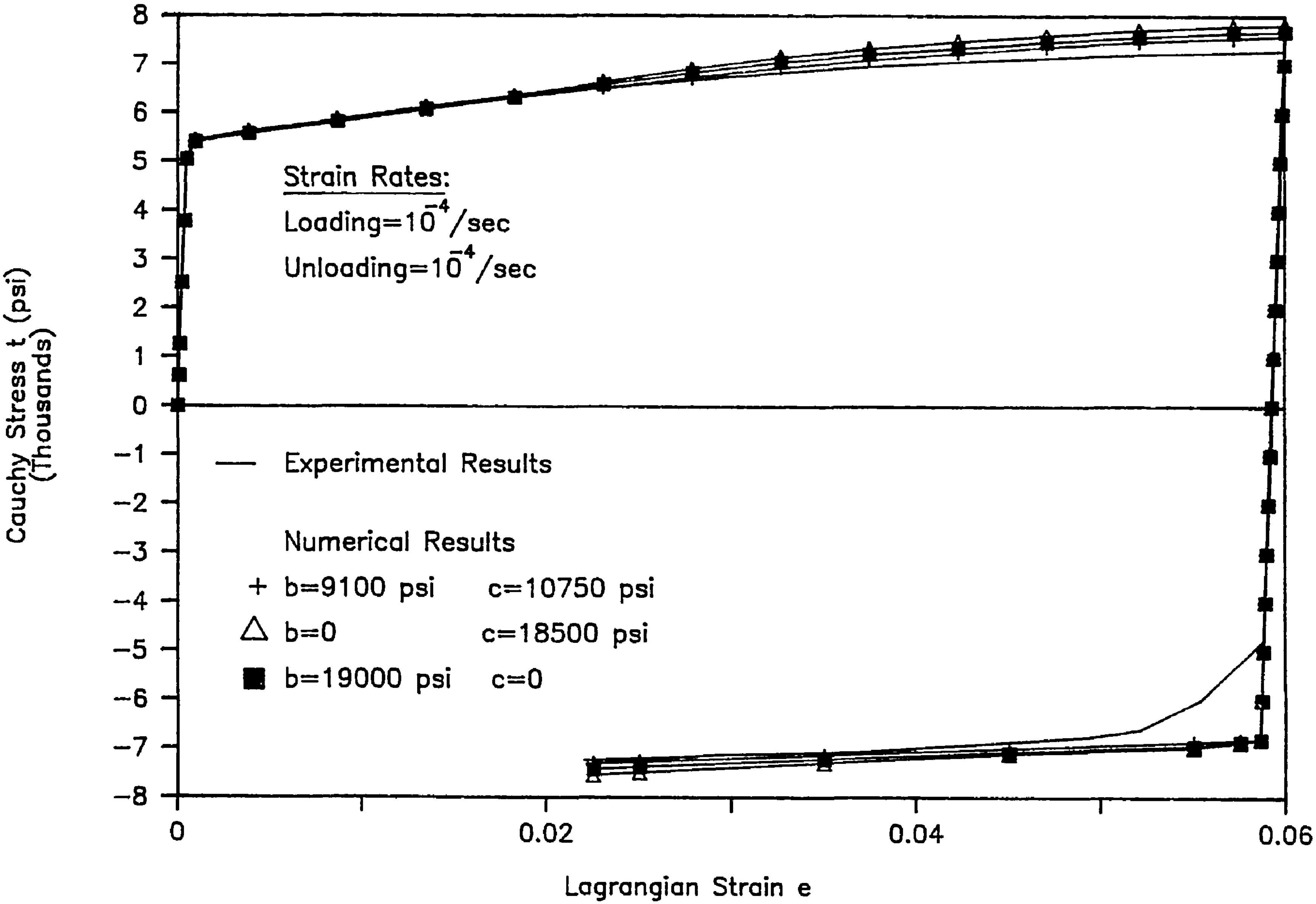

Figure 19. Experimental & Theoretical Relations Between e_{11} & t_{11}

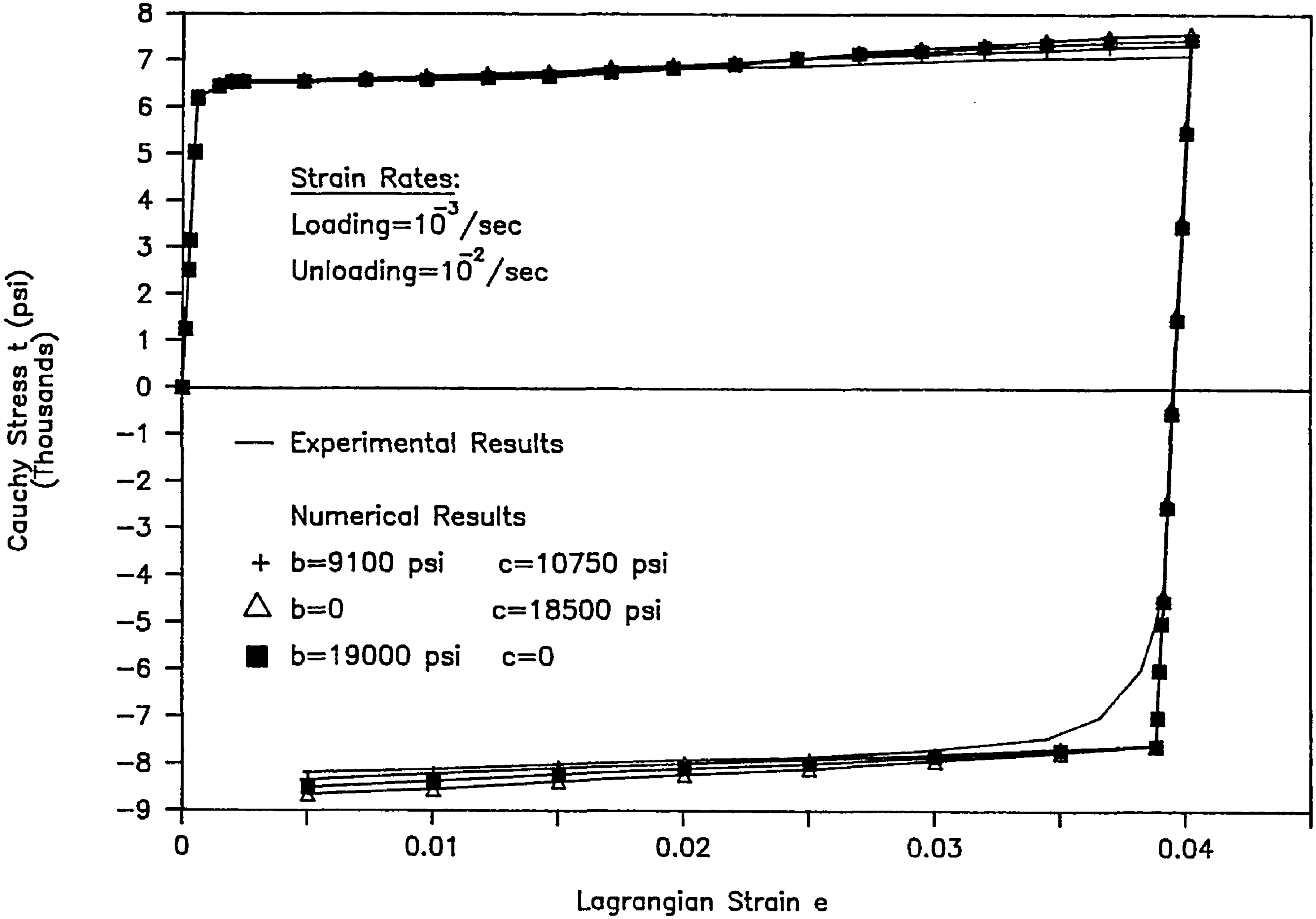

Figure 20. Experimental & Theoretical Relations Between e_{11} & t_{11}

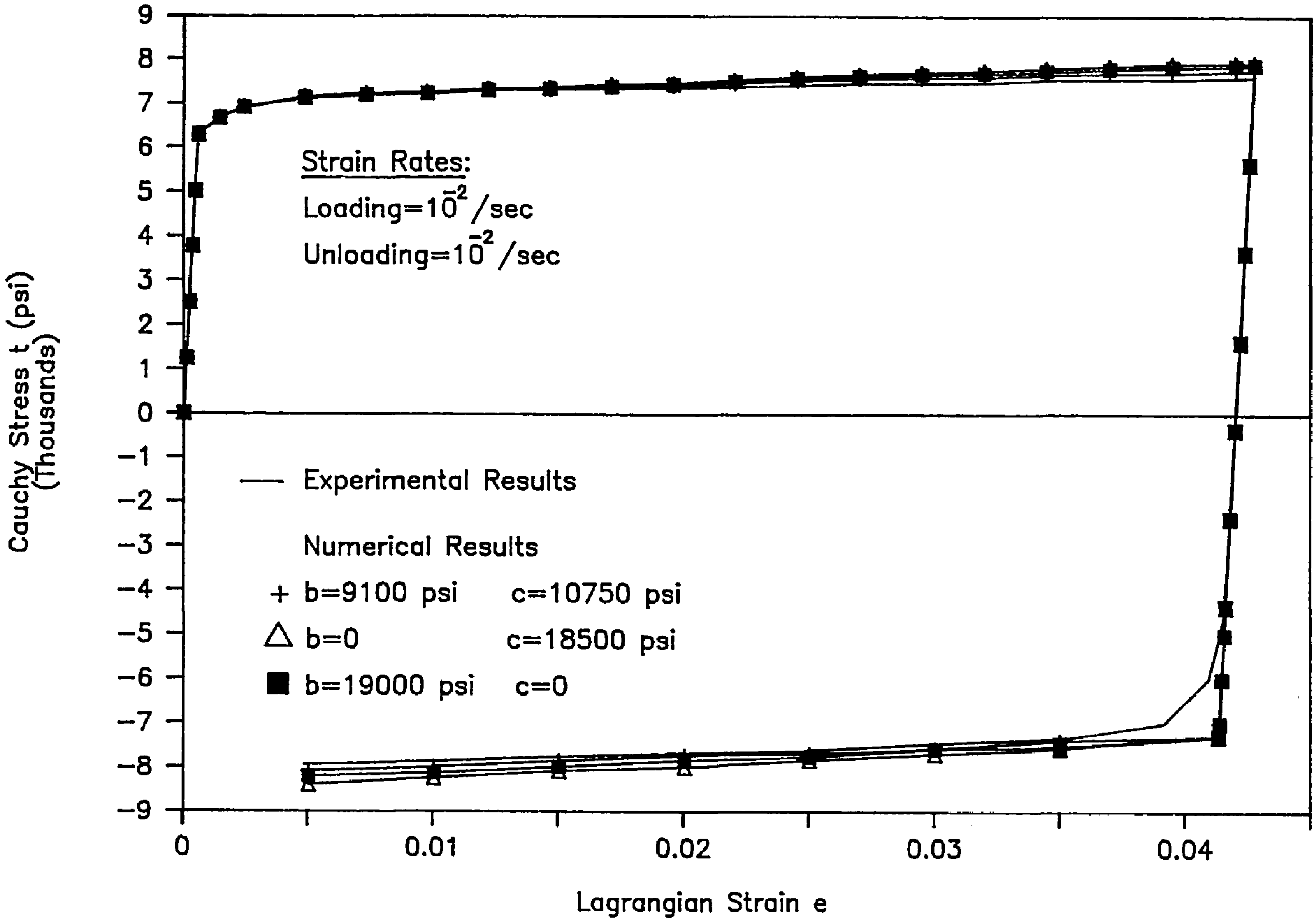

Figure 21. Experimental & Theoretical Relations Between e_{11} & t_{11}

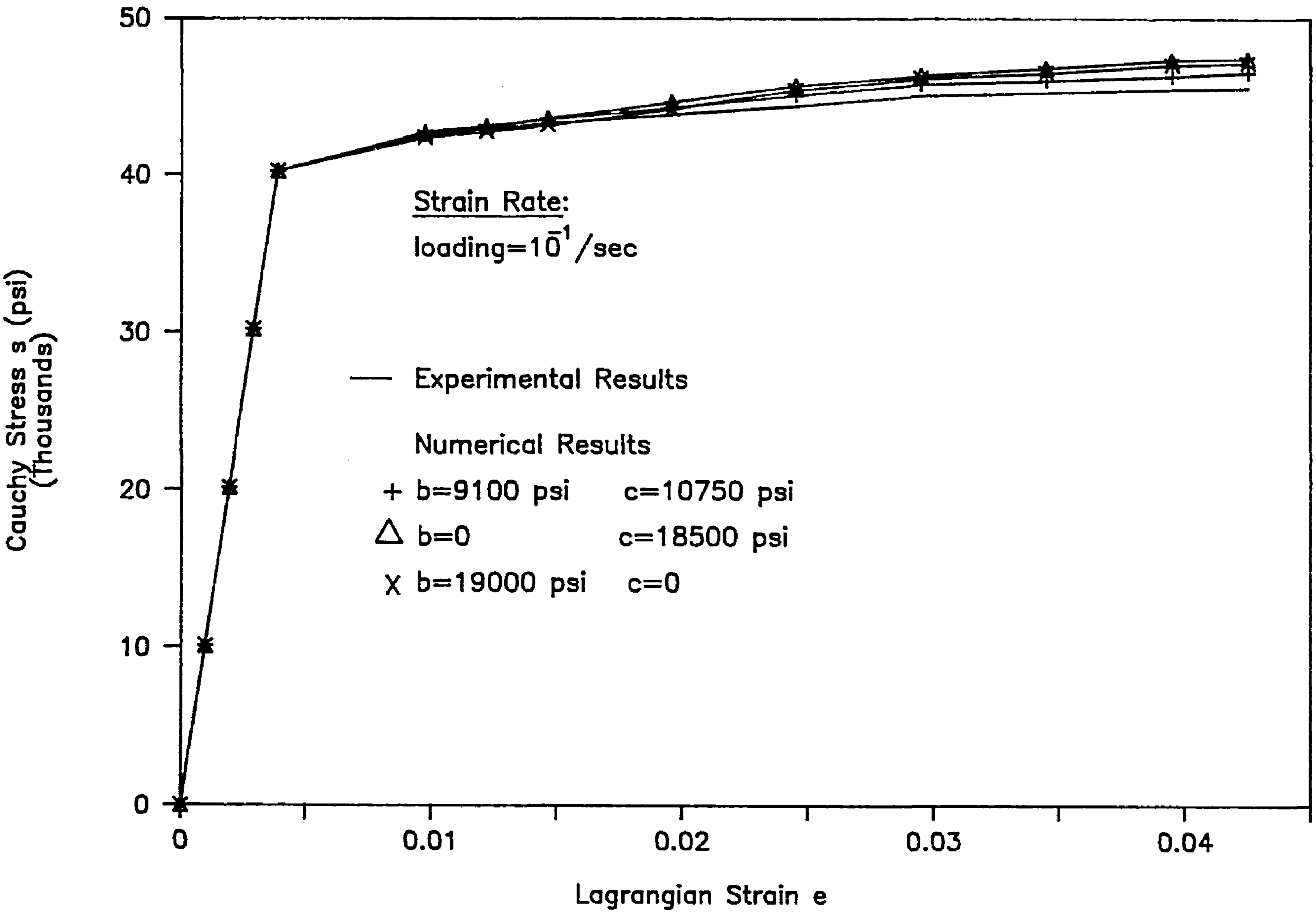

Figure 22. Experimental & Theoretical Relations Between e_{11} & t_{11}

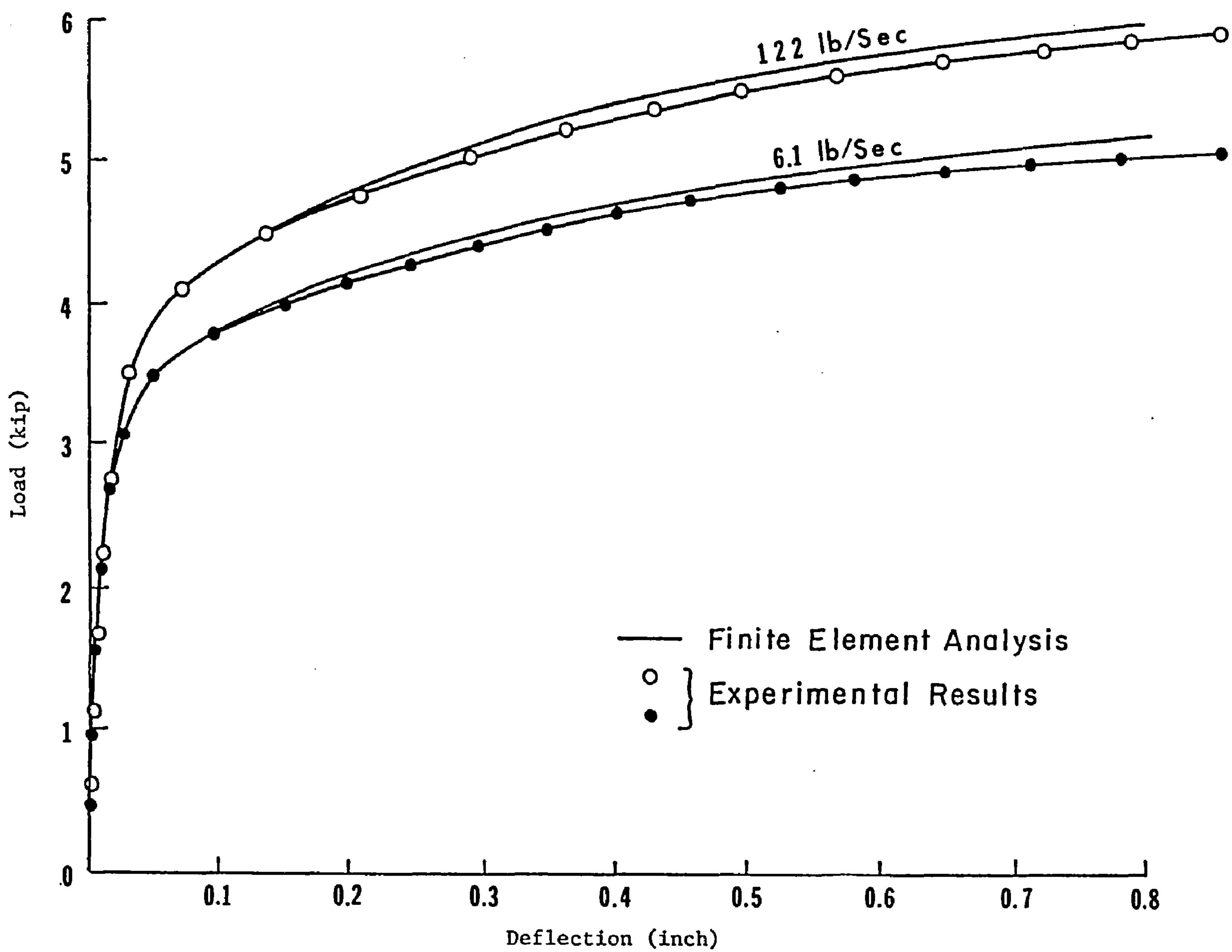

Figure 23. Deflection at the Center (Bottom) of the Plate

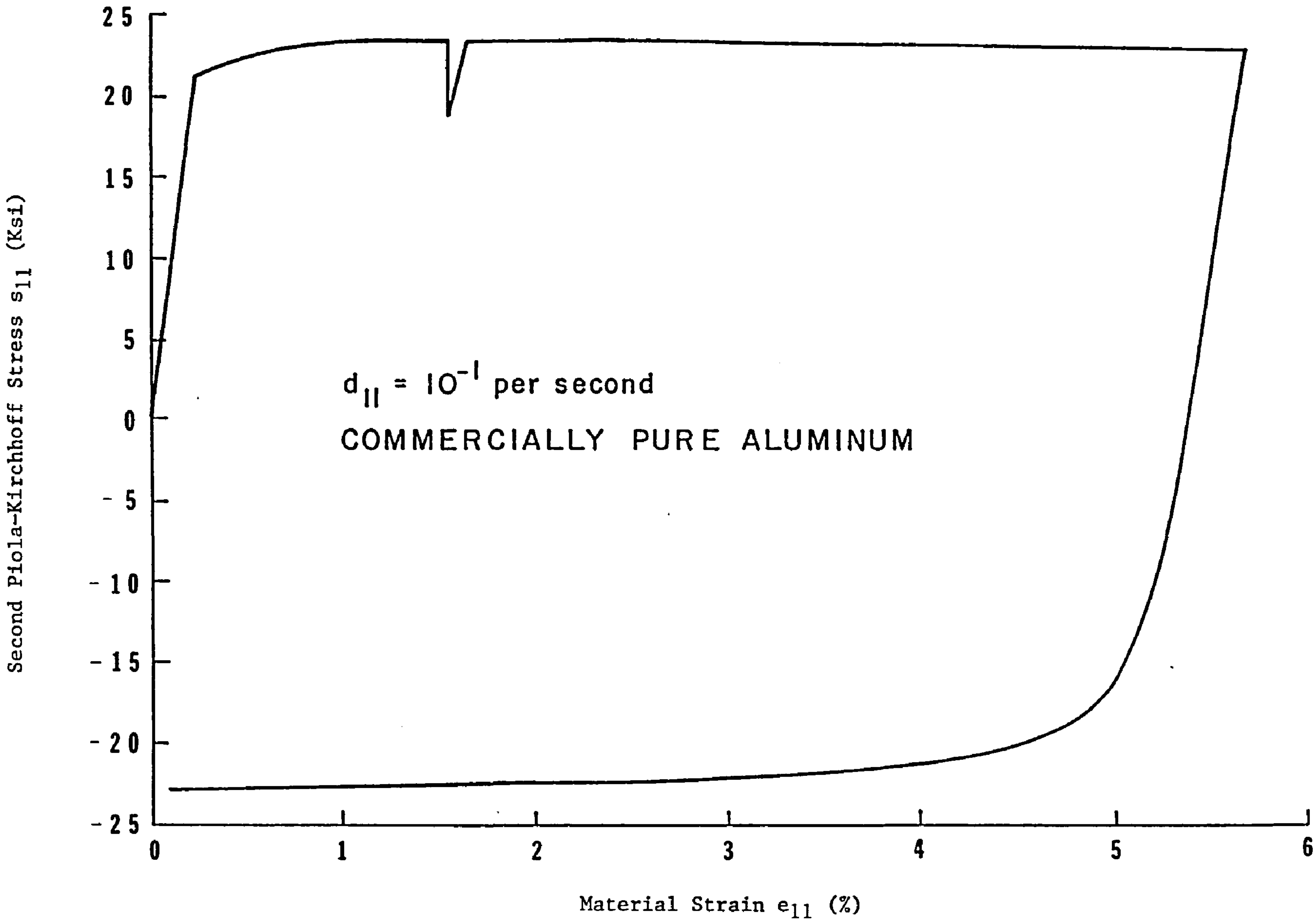

Figure 24. Material Strain e_{11} vs. Second Piola-Kirchhoff Stress s_{11} at Constant Spatial Strain Rate d_{11} with the Relaxation Period Indicated by the Vertical Drop

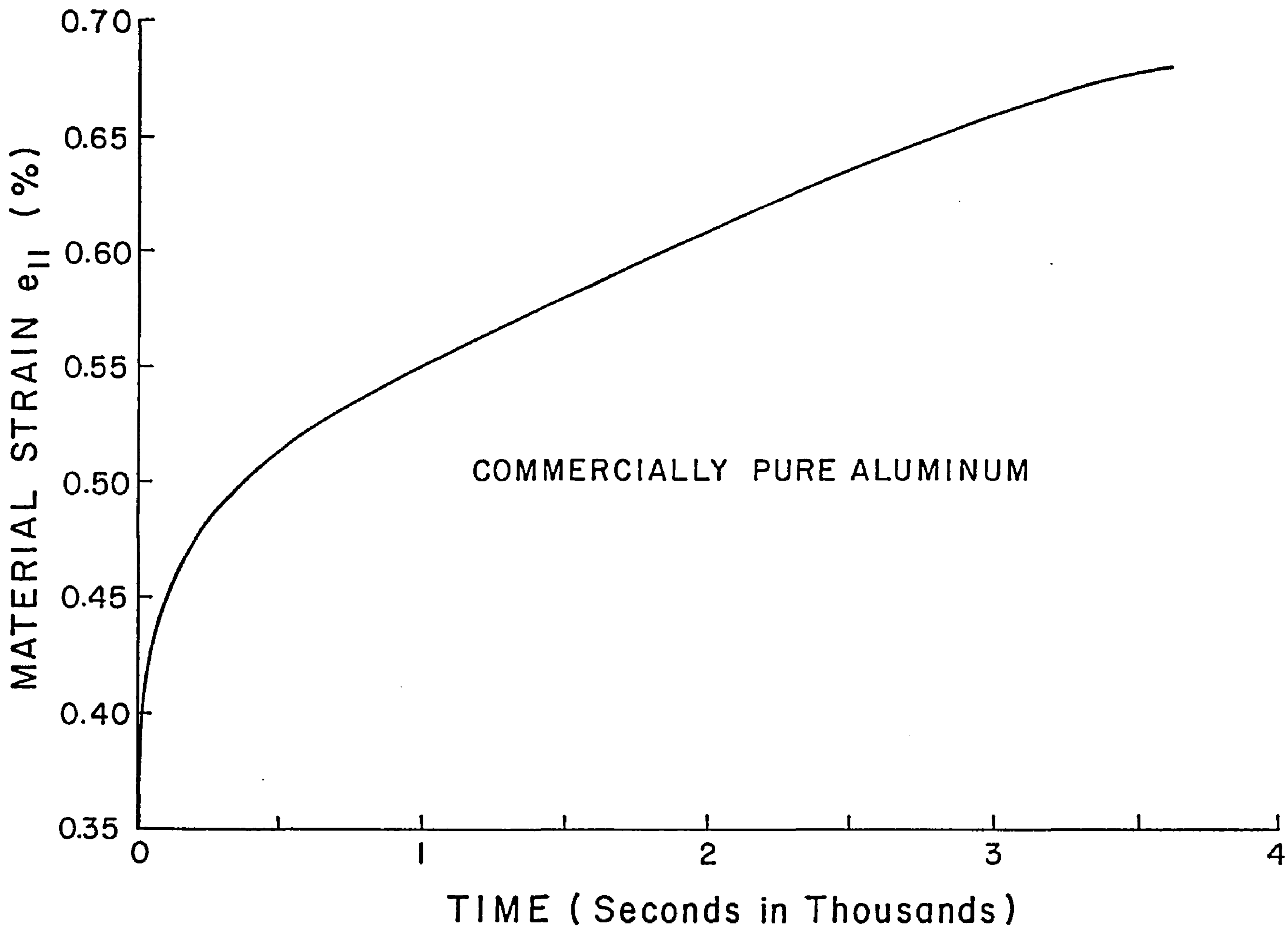

FIGURE 25. CREEP STRAIN AT A UNIAXIAL STRESS OF **22113 PSI**

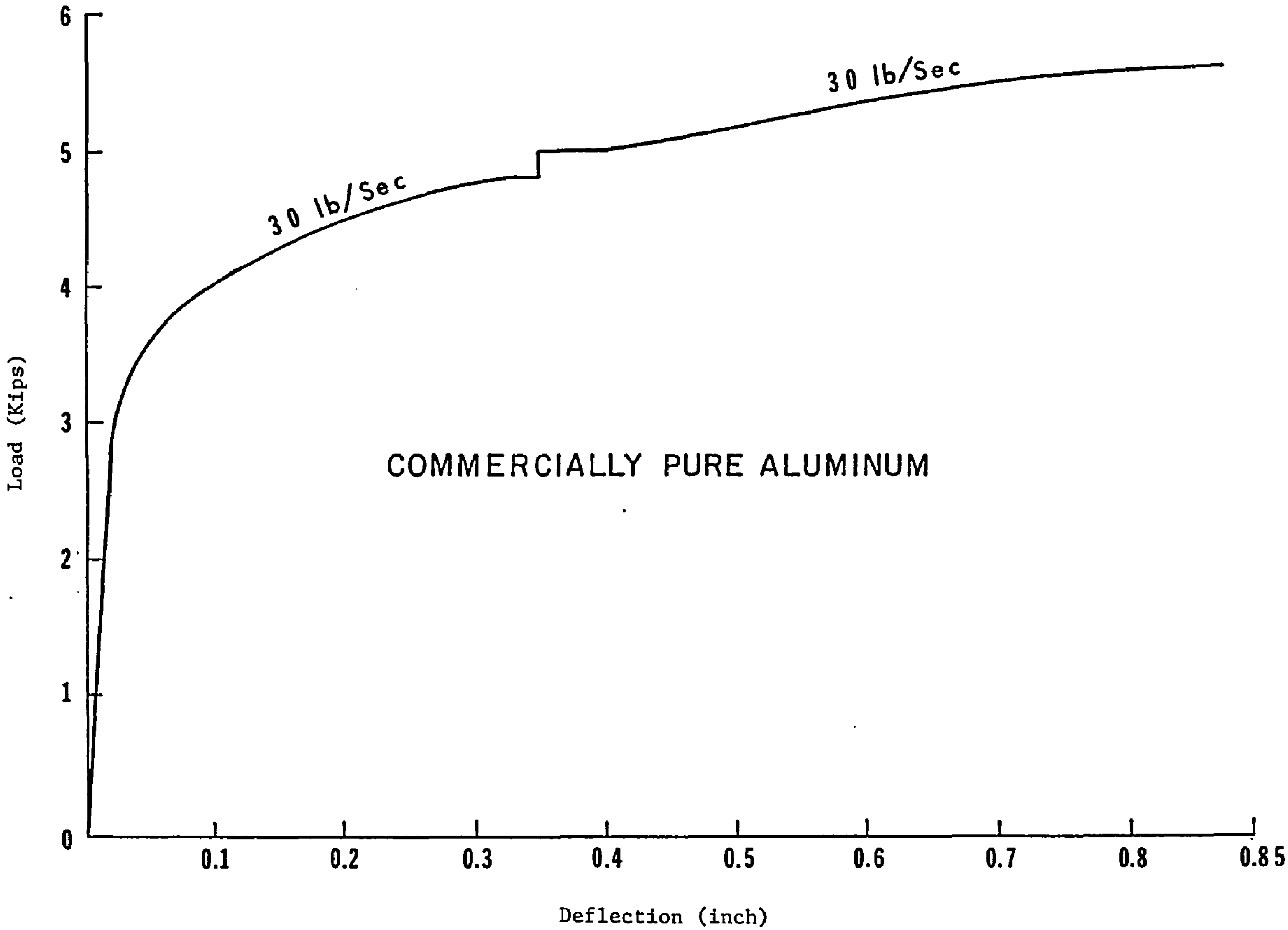

Figure 26. Load-Deflection Curve with Creep Effect

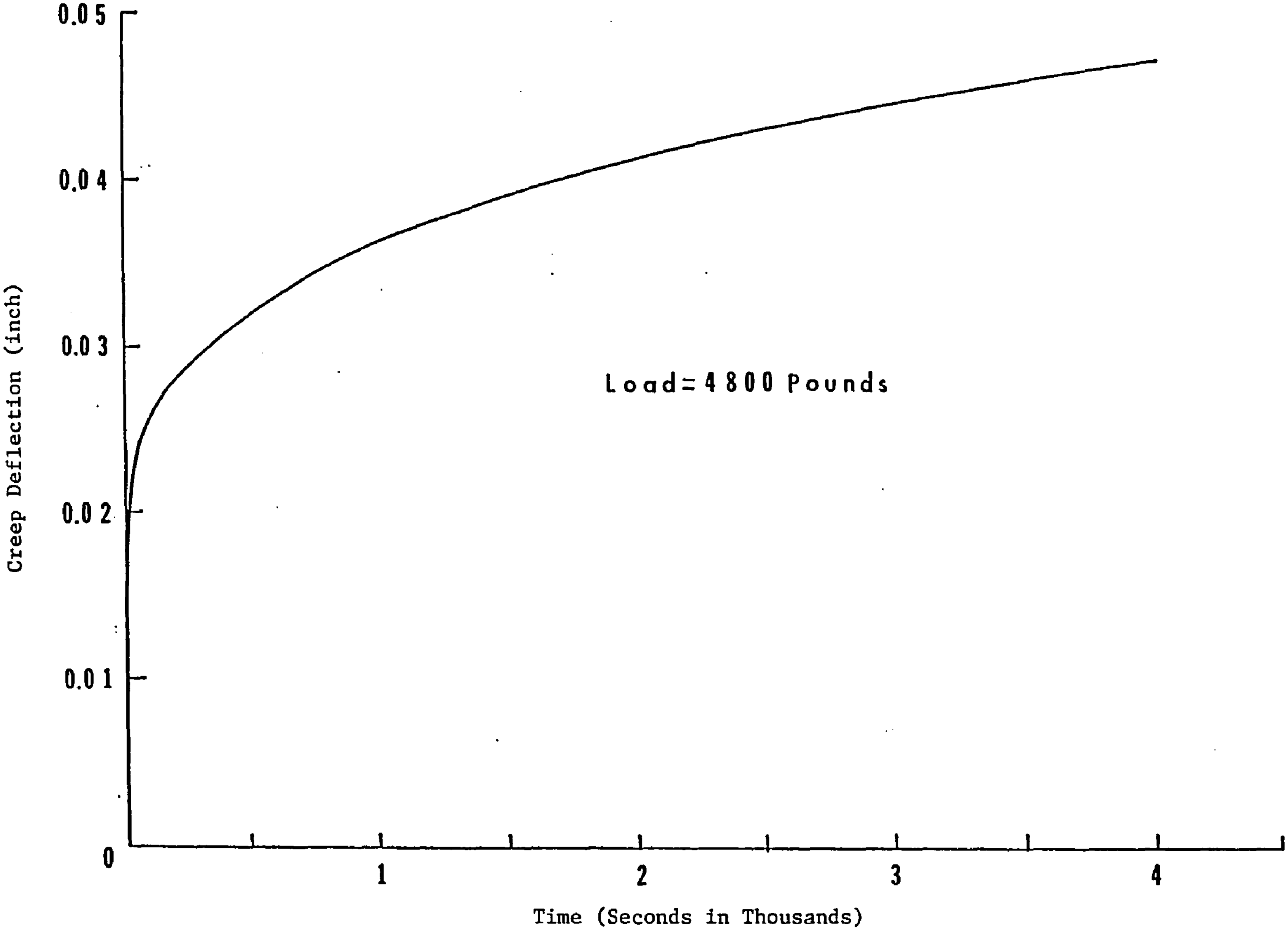

Figure 27. Creep Deflection for the Plate Problem

Chapter 7

SUMMARY AND CONCLUSION

A constitutive model for elastic/viscoplastic material behavior at finite strains is presented. The model is linear elastic before the inelastic state is reached and become elastic/viscoplastic after the plastic state has been exceeded. The inelastic strain rate tensor is assumed to be normal to each point of the rate dependent convex yield surface and the hardening effect is due to both isotropic and anisotropic work hardening.

The uniaxial stress-strain curves obtained experimentally in this work are numerically simulated through the proposed constitutive model. The model is anticipated to predict the material behavior at both finite strains and high strain rates. Nevertheless, the experiments conducted in this work are limited to a maximum of 6% strain and 10^{-1} per second strain rate.

Due to the confinement in the current experimental work to uniaxial tests, Ziegler's kinematic hardening rule is used. The yield surface is limited in this work to kinematic hardening and isotropic deformation (expansion or contraction) without distortion of the shape of the surface. A more realistic deformation of the yield surface proposed by Eisenberg and Yen [14] will require biaxial testing. This will obviously increase the number of material parameters for the description of the model, and will also require extensive and more elaborate experimental work.

The use of the Lagrangian reference frame in this work enables us to bypass the use of the stress rate in the Eulerian reference frame.

In this way, it bypasses the problem of the correct identification of a proper stress rate in the Eulerian reference frame. Furthermore, the use of the Lagrangian stress rate as an objective stress rate helps one utilize the numerical algorithms used for small strain viscoplasticity. This is because both the Lagrangian stress rate and the stress rate for small strains have identical time derivative operators.

Both the dynamic yield condition (a von Mises type) and the flow rule in this work are defined in the Eulerian reference frame and then properly transformed to the Lagrangian reference frame. This approach preserves the accuracy of the interpretation of the material behavior in the Eulerian frame (Voyiadjis [34]). Although Ziegler's hardening rule in this work is defined directly into the Lagrangian reference frame, nevertheless, it has identical form when transformed into the Eulerian reference frame. This is provided the Truesdell-Oldroyd stress rate is used as an objective stress rate in the Eulerian reference frame (Voyiadjis [46]).

This work also demonstrates the effectiveness of the proposed viscoplastic constitutive model in solving complex problems (i.e., any shape and any deformation). The evaluation of the proposed constitutive equations is determined by comparison of the experimental load deflection curve at different loading rates for the bending of a moderately thick plate with the corresponding finite element solution for the same problem. The general shape of the load deflection curves are in agreement as well as the corresponding values. The suggested viscoplastic model complies with experimental evidence. Strains of the order of 25% were exhibited in the plate bending problem.

The present constitutive formulation does not provide the capability of computing the creep strain.

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Appendix A

```
C ******************************************************************
C
C    PROGRAM:   S A V P A
C
C    THIS PROGRAM COPMUTES THE STRESS INCREMENT, AND TOTAL STRESSES
C    DUE TO AN INCREMENT OF STRAINS FOR A UNIAXIAL STATE OF STRESS.
C
C   ****************************************************************
C
      PROGRAM MAIN
C
C   DELET(I,J)  = TOTAL STRAIN INCREMENT
C   DELEE(I,J)  = ELASTIC STRAIN INCREMENT
C   DELEVP(I,J) = VISCO-PLASTIC STRAIN INCREMENT
C   DELS(I,J)   = STRESS INCREMENT
C   DELT        = INCREMENTAL TIME IN SEC.
C   DELA(I,J)   = INCREMENT OF LAGRANGIAN SHIFT TENSOR
C   A1,A2,A3    = CONSTANTS DETERMINED EXPERIMENTALLY
C   S0          = STATIC ONE-DIMENSIONAL YIELD STRESS
C   SL          = LIMITING ONE-DIMENSIONAL STRESS
C   THETA       = CONSTANT EQUAL TO 0.5
C   Y           = YOUNG'S MODULUS
C   IFLAG1      = 1 FOR PLANE STRAIN
C               = 2 FOR PLANE STRESS
C               = 3 FOR PLANE STRESS
C
C     --------------------------------------------------------------
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      COMMON/M0/S(3,3),C(3,3),A(3,3),E(3,3)
      COMMON/M1/AJAC,AK,C0,AKAPPA
      COMMON/M2/F,H,H9
      COMMON/M3/SC1,SC2,SC3,SC4,SC5,SC6,SC7,SC8,SC9,SC10,SC11,SC12,
     .          SC13,SC14,SC15,SC16,SC17,SC18,SC19
      COMMON/M4/AC1,AC2,AC3,AC4,AC5,AC6,AC7,AC8,AC9,AC10,AC11,AC12,
     .          AC13,AC14,AC15,AC16,AC17,AC18,AC19
      COMMON/M5/A1,A2,A3,SL,S0
      COMMON/M6/AMMA,PGAMF
      COMMON/M7/PFS11,PFS22,PFS33,PFS12,PSAFSA,PSBFSA,PSDFSA,
     .          PSBFSB,PSDFSB,PSDFSD,PSCFSA,PSCFSB,PSCFSD,PSCFSC
      COMMON/M8/Q(4,4)
      COMMON/M9/COMP(4,4),AMU,Y,DELT,THETA,G,ALMDA
      COMMON/M10/D(4,4)
      COMMON/M12/DELEE(3,3),DELEVP(3,3),DELET(3,3),DELA(3,3),DELS(3,3)
      COMMON/M13/D1(4,4)
      COMMON/M14/IFLAG1
      COMMON/M15/B(4,4)
      COMMON/INPUT1/B0,C0,Y,AMU,G,A1,A2,A3,SL,S0,AK0,AK,AKAPPA,DELT,
```

```
     .             THETA,IFLAG1,INC,DELT(2,2),S(2,2),E(2,2)
C
      CALL INPUT
C
      DIMENSION STRESS(3,3,600),STRAIN(3,3,600)
C
C     COMPUTE E(1,1) AND E(3,3)
C
      E(1,1) = -ALMDA*E(2,2)/(2.*(ALMDA+G))
      E(3,3) = E(1,1)
C
      WRITE(6,106)
C
C     CALCULATE J, GREEN'S TENSOR C(I,J)
C
    1 CALL AJACOB
C
C     CALCULATE  YIELD FUNCTION F
C
      CALL YIELDF
C
C     CALCULATE GAMMA
C
      CALL GAMMA
C
C     CALCULATE PARTIAL DERIVATIVE OF F WRT S(I,J) AND
C     PARTIAL WRT S(I,J) OF THE PFWRTS
C
      CALL PFWRTS
C
C     COMPUTE VISCO-PLASTIC STRAIN INCREMENT
C
      DELEVP(2,2)=AJAC*AMMA*F*PFS22
C
C     COMPUTE THE ELASTIC COMPONENT OF DELET(2,2)
C
      DELEE(2,2)= DELET(2,2) - DELEVP(2,2)
C
C     COMPUTE THE ELASTIC COMPONENT OF DELET(1,1)
C
      DELEE(1,1)=-ALMDA*DELEE(2,2)/(2.0*(ALMDA+G))
      DELEE(3,3)=DELEE(1,1)
C
C     COMPUTE THE VISCO-PLASTIC COMPONENT OF DELET(1,1)
C
      DELEVP(1,1)=-((2.0*E(1,1)+1.0)*DELEVP(2,2))/(2.*(2.*E(2,2)+1.0))
      DELEVP(3,3)=DELEVP(1,1)
C
C     COMPUTE DELET(1,1) AND DELET(3,3)
C
      DELET(1,1) = DELEE(1,1) + DELEVP(1,1)
      DELET(3,3) = DELET(1,1)
```

```
C
C     COMPUTE PARTIAL DERIVATIVE OF VP STRAIN RATE WRT STRESS
C     AND BUILD THE ( Q ) MATRIX
C
      CALL PEDWTS
C
C     COMPUTE [D] .. [D]=[ [C]+DELT*THETA*[Q] ]
C
      CALL DEVP
      CALL ENVERS
      CALL SOLVE
C
C     CALCULATE SHIFT TENSOR
C
      AN1 =AJAC*AMMA*F*B0
      AN2 =PFS11**2+PFS22**2+PFS33**2
      DEN =(S(2,2)-A(2,2))*PFS22
      AMUDT =AN1*AN2/DEN
      DELA(2,2) =(S(2,2)-A(2,2))*AMUDT
C
C     CALCULATE THE PLASTIC WORK DONE, AKAPPA, AND THE TOTAL
C     SHIFT TENSOR ( A )
C
      AKAPPA=AKAPPA+S(2,2)*DELEVP(2,2)/AJAC
      A(2,2)=A(2,2) + DELA(2,2)
C
C     COMPUTE TOTAL STRAINS
C
      E(1,1)=E(1,1)+DELET(1,1)
      E(2,2)=E(2,2)+DELET(2,2)
      E(3,3)=E(1,1)
      E(1,2)=E(1,2)+DELET(1,2)
C
C     PLACE TOTAL STRESS S(I,J) IN STREE(I,J,INC)
C
      DO 103 I=1,3
      STRESS(I,I,INC)=S(I,I)
      STRAIN(I,I,INC)=E(I,I)
  103 CONTINUE
      STRESS(1,2,INC)=S(1,2)
      STRAIN(1,2,INC)=E(1,2)
C
      WRITE(6,107) INC,(STRESS(I,I,INC),I=1,3),(E(I,I),I=1,3),
     .             (DELS(I,I),I=1,3)
C
  105 FORMAT(/,' MAIN STRESS(I,J,INC)',5X,4E15.7,/)
  106 FORMAT(1H1,/,2X,'INC',2X,'STRESS 11',2X,'STRESS 22',2X,'STRESS33'
     1,2X,' E 11 ',2X,' E 22 ',2X,' E 33 ',2X,' DEL S11 ',2X,' DEL S22'
     2,2X,' DEL S33 ',/)
  107 FORMAT(I5,2X,F9.0,2X,F9.0,2X,F9.0,1X,F8.6,1X,F8.6,1X
     1,F7.5,1X,F7.1,4X,F7.1,4X,F7.1
      IF(INC.EQ.490) GO TO 2
```

```
      INC=INC+1
C
C     LOOP FOR  NEXT INCREMENT
C
      GO TO 1
    2 WRITE(6,110)
      WRITE(6,111) STRESS(2,2,1),STRAIN(2,2,1)
      DO 109 J=10,INC,24
      WRITE(6,108) J,STRESS(2,2,J),STRAIN(2,2,J)
  109 CONTINUE
  108 FORMAT(I5,F6.0,1X,F9.6)
  110 FORMAT(1H1)
  111 FORMAT(F6.0,1X,F9.6)
      STOP
      END
C
C ******************************************************************
C
      SUBROUTINE INPUT
C
C
C ------------------------------------------------------------------
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      COMMON/INPUT1/B0,C0,Y,AMU,G,A1,A2,A3,SL,S0,AK0,AK,AKAPPA
     .              DELT,THETA,IFLAG1,INC,DELT(2,2),S(2,2),E(2,2)
C
C     INITIALZE
C
      DO 500 I =1,3
      DO 500 J =1,3
      S(I,J) =0.0
      C(I,J) =0.0
      A(I,J) =0.0
      E(I,J) =0.0
      DELS(I,J)=0.0
      DELEE(I,J)=0.0
      DELA(I,J)=0.0
      DELET(I,J)=0.0
      DELEVP(I,J)=0.0
500   CONTINUE
      AKAPPA=0.0
C
C     INPUT MATERIAL PROPERTIES
C
      B0=9000.0
      C0=10500.0
      Y=10.4E+6
      AMU=0.25
      ALMDA=Y*AMU/((1.+AMU)*(1.-2.*AMU))
      G=Y/(2.*(1.+AMU))
```

```
      A1=11.0
      A2=0.33
      A3=0.080
      SL=36360.0
      S0=5000.
      AK0=5049.0
      AK=AK0/SQRT(3.)
      AKAPPA=0.0
      DELT=1.0
      THETA=0.50
      IFLAG1=3
      INC=1
      DELET(2,2)=1.0E-04
C
C     INPUT YIELD STRESS AND STRAIN (S22&E22) FOR A GIVEN STRIAN RATE
C
      S(2,2) = AK0
      E(2,2) = AK0/Y
      RETURN
      END
C
C ------------------------------------------------------------------
C
      SUBROUTINE AJACOB
C THIS SUBROUTINE EVALUATES GREEN'S TENSOR C(I,J) AND JACOBIAN  J
C
C ------------------------------------------------------------------
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      COMMON/M0/S(3,3),C(3,3),A(3,3),E(3,3)
      COMMON/M1/AJAC,AK,C0,AKAPPA
C
C
C     CALCULATE GREEN DEFORMATION TENSOR
C
      C(1,1)=2.*E(1,1)+1.
      C(2,2)=2.*E(2,2)+1.
      C(3,3)=2.*E(3,3)+1.
      C(1,2)=2.*E(1,2)
C
C     CALCULATE THE JACOBIAN
C
      EINV1=E(1,1)+E(2,2)+E(3,3)
      EINV2=(E(1,1)*E(2,2))+(E(2,2)*E(3,3))+(E(1,1)*E(3,3))-(E(1,2)**2)
      EINV3=(E(1,1)*E(2,2)*E(3,3))-(E(3,3)*E(1,2)*E(1,2))
C
      AJAC1=1.+(2.*EINV1)+(4.*EINV2)+(8.*EINV3)
      AJAC=AJAC1**(0.5)
      RETURN
      END
C
```

```
C **********************************************************************
C
      SUBROUTINE YIELDF
C
C THIS SUBROUTINE EVALUATES THE DYNAMIC YIELD FUNCTION, F
C
C S(I,J)= SECOND PIOLA-KIRCHHOFF STRESS TENSOR
C A(I,J)= SHIFT STRESS TENSOR (LAGRANGIAN COORDINATE)
C C(I,J)= GREEN'S DEFORMATION TENSOR (DISPLACEMENT GRADIENT TENSOR)
C AJAC..= DETERMINANT OF THE JACOBIAN TRANSFORMATION ZK TO XK
C AK ....= INITIAL YIELD STRESS
C C0 ...= CONSTANT THAT DESCRIBES THE ISOTROPIC COMPONENT OF HARDENING
C AKAPPA = PLASTIC WORK
C EINV1. = FIRST STRAIN INVARIENT
C EINV2. = SECOND STRIAN INVARIENT
C EINV3. = THIRD STRIAN INVARIENT
C
C ----------------------------------------------------------------------
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      COMMON/M0/S(3,3),C(3,3),A(3,3),E(3,3)
      COMMON/M1/AJAC,AK,C0,AKAPPA
      COMMON/M2/F,H,H9
      COMMON/M3/SC1,SC2,SC3,SC4,SC5,SC6,SC7,SC8,SC9,SC10,SC11,SC12,
     .          SC13,SC14,SC15,SC16,SC17,SC18,SC19
      COMMON/M4/AC1,AC2,AC3,AC4,AC5,AC6,AC7,AC8,AC9,AC10,AC11,AC12,
     .          AC13,AC14,AC15,AC16,AC17,AC18,AC19
C
      SC1=S(1,1)*C(1,1)*C(1,1)
      SC2=S(1,1)*C(1,2)*C(1,2)
      SC3=S(1,1)*C(1,1)*C(2,2)
      SC4=S(1,1)*C(1,1)*C(1,2)
      SC5=S(1,1)*C(1,1)*C(3,3)
C
      SC6=S(2,2)*C(1,2)*C(1,2)
      SC7=S(2,2)*C(1,1)*C(2,2)
      SC8=S(2,2)*C(2,2)*C(2,2)
      SC9=S(2,2)*C(2,2)*C(1,2)
      SC10=S(2,2)*C(2,2)*C(3,3)
C
      SC11=S(3,3)*C(1,1)*C(3,3)
      SC12=S(3,3)*C(2,2)*C(3,3)
      SC13=S(3,3)*C(3,3)*C(1,2)
      SC14=S(3,3)*C(3,3)*C(3,3)
C
      SC15=S(1,2)*C(1,1)*C(1,2)
      SC16=S(1,2)*C(2,2)*C(1,2)
      SC17=S(1,2)*C(1,1)*C(2,2)
      SC18=S(1,2)*C(1,2)*C(1,2)
      SC19=S(1,2)*C(3,3)*C(1,2)
C
```

```
      AC1=A(1,1)*C(1,1)*C(1,1)
      AC2=A(1,1)*C(1,2)*C(1,2)
      AC3=A(1,1)*C(1,1)*C(2,2)
      AC4=A(1,1)*C(1,1)*C(1,2)
      AC5=A(1,1)*C(1,1)*C(3,3)
C
      AC6=A(2,2)*C(1,2)*C(1,2)
      AC7=A(2,2)*C(1,1)*C(2,2)
      AC8=A(2,2)*C(2,2)*C(2,2)
      AC9=A(2,2)*C(2,2)*C(1,2)
      AC10=A(2,2)*C(2,2)*C(3,3)
C
      AC11=A(3,3)*C(1,1)*C(3,3)
      AC12=A(3,3)*C(2,2)*C(3,3)
      AC13=A(3,3)*C(3,3)*C(1,2)
      AC14=A(3,3)*C(3,3)*C(3,3)
C
      AC15=A(1,2)*C(1,1)*C(1,2)
      AC16=A(1,2)*C(2,2)*C(1,2)
      AC17=A(1,2)*C(1,1)*C(2,2)
      AC18=A(1,2)*C(1,2)*C(1,2)
      AC19=A(1,2)*C(3,3)*C(1,2)
C
C     START BUILDING THE DYNAMIC YIELD FUNCTION, F
C
      H1=((S(1,1)*SC1)/3.) + (S(1,1)*SC6) + ((2.*S(1,1)*SC15)/3.) +
     .   ((S(2,2)*SC8)/3.) + (S(3,3)*SC14/3.)+ ((2.*S(2,2)*SC16)/3.)
      H2=((S(1,2)*SC17)/2.)- ((S(1,2)*SC18)/6.) - ((S(1,1)*SC7)/3.) -
     .   ((S(1,1)*SC11)/3.)- ((S(2,2)*SC12)/3.) - ((S(3,3)*SC19)/3.)
      H3=S(1,1)*((-2.*AC1/3.)-(AC6)-(2.*AC15/3.)+(AC7/3.)+(AC11/3.))
      H4=S(2,2)*(-AC2-(2.*AC8/3.)-(2.*AC16/3.)+(AC3/3.)+(AC12/3.))
      H5=S(3,3)*(-(2.*AC14/3.)+(AC5/3.)+(AC10/3.)+(AC19/3.))
      H6=S(1,2)*(-(2.*AC4/3.)-(2.*AC9/3.)-AC17+(AC13/3.)+(AC18/3.))
      H7=((A(1,1)*AC1)/3.) + (A(1,1)*AC6) + ((2.*A(1,1)*AC15)/3.) +
     .   ((A(2,2)*AC8)/3.) + (A(3,3)*AC14/3.)+ ((2.*A(2,2)*AC16)/3.)
      H8=((A(1,2)*AC17)/2.)- ((A(1,2)*AC18)/6.) - ((A(1,1)*AC7)/3.) -
     .   ((A(1,1)*AC11)/3.)- ((A(2,2)*AC12)/3.) - ((A(3,3)*AC19)/3.)
C
      H=H1+H2+H3+H4+H5+H6+H7+H8
C
      H9=(AJAC*AJAC)*((AK*AK)+(C0*AKAPPA))
      H10=1.0/H9
      F=SQRT(H10*H)-1.0
C
      RETURN
      END
C
C *******************************************************************
C
      SUBROUTINE GAMMA
C
C THIS SUBROUTINE EVALUATE THE MATERIAL FUNCTION GAMMA(F)
```

```
C                    A1
C     GAMMA =--------------
C             F**A2(FL-F)**A3
C
C ALSO, THIS SUBROUTINE EVALUATES THE PARTIAL DERIVATIVE OF GAMMA
C W.R.T. THE DYNAMIC YIELD FUNCTION F
C
C WHERE:
C     A1,A2,A3: CONSTANTS DETERMINED EXPERIMENTALY
C     FL...= ((SL/S0)**2.) - 1.0
C     SL...= LIMITING ONE DIMENTIONAL STRESS OBTAINED FROM DYNAMIC
C            TESTS AT THE HIGHEST STRAIN RATES OF INTEREST
C     S0...= STATIC ONE-DIMENSTIONAL YIELD STRESS
C     PGAMF= PARTIAL DERIVATIVE OF GAMMA W.R.T. F
C
C ---------------------------------------------------------------------
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      COMMON/M2/F,H,H9
      COMMON/M5/A1,A2,A3,SL,S0
      COMMON/M6/AMMA,PGAMF
C
      FL=((SL/S0)**2.) - 1.
      FA2=F**A2
      FA2P1=F**(A2+1.0)
      FLFA3=(FL-F)**A3
      FLFA31=(FL-F)**(A3+1)
      AMMA=A1/(FA2*FLFA3)
      PGAMF=((A1*A3)/(FA2*FLFA31))-((A1*A2)/(FA2P1*FLFA3))
C
      RETURN
      END
C
C ********************************************************************
C
      SUBROUTINE PFWRTS
C
C THIS SUBROUTINE EVALUATES THE PARTIAL DERIVATIVE OF F W.R.T.
C THE STRESS (S11,S22,S33,S12) AND THE PARTIAL DERIVATIVE
C W.R.T. THE STRESS (S11,S22,S33,S12) OF THE PARTIAL DERIVATIVE
C OF F W.R.T. S
C
C PFSIJ...= PARTIAL DERIVATIVE OF F W.R.T. SIJ
C PSAFSA..= PARTIAL DERIVATIVE W.R.T. S11 OF THE PFS11
C PSAFSB..= PARTIAL DERIVATIVE W.R.T. S11 OF THE PFS22
C PSAFSC..= PARTIAL DERIVATIVE W.R.T. S11 OF THE PFS33
C PSAFSD..= PARTIAL DERIVATIVE W.R.T. S11 OF THE PFS12
C
C ---------------------------------------------------------------------
C
      IMPLICIT REAL*8(A-H,O-Z)
```

```
C
      COMMON/M0/S(3,3),C(3,3),A(3,3),E(3,3)
      COMMON/M2/F,H,H9
      COMMON/M3/SC1,SC2,SC3,SC4,SC5,SC6,SC7,SC8,SC9,SC10,SC11,SC12,
     .         SC13,SC14,SC15,SC16,SC17,SC18,SC19
      COMMON/M4/AC1,AC2,AC3,AC4,AC5,AC6,AC7,AC8,AC9,AC10,AC11,AC12,
     .         AC13,AC14,AC15,AC16,AC17,AC18,AC19
      COMMON/M7/PFS11,PFS22,PFS33,PFS12,PSAFSA,PSBFSA,PSDFSA,
     .         PSBFSB,PSDFSB,PSDFSD,PSCFSA,PSCFSB,PSCFSD,PSCFSC
C
      A4=0.5*SQRT(1/(H9*H))
C
      H11=(2.*SC1/3.)+SC6+(2.*SC15/3.)-(SC7/3.)-(SC11/3.)
     .   -(2.*AC1/3.)-AC6-(2.*AC15/3.)+(AC7/3.)+(AC11/3.)
      H12=SC2+(2.*SC8/3.)+(2.*SC16/3.)-(SC3/3.)-(SC12/3.)
     .   -AC2-(2.*AC8/3.)-(2.*AC16/3.)+(AC3/3.)+(AC12/3.)
      H13=(2.*SC4/3.)+(2.*SC9/3.)+SC17-(SC18/3.)-(SC13/3.)
     .   -(2.*AC4/3.)-(2.*AC9/3.)-AC17+(AC18/3.)+(AC13/3.)
      H14=(2.*SC14/3.)-(SC5/3.)-(SC10/3.)-(SC19/3.)
     .   -(2.*AC14/3.)+(AC5/3.)+(AC10/3.)+(AC19/3.)
C
      PFS11=A4*H11
      PFS22=A4*H12
      PFS33=A4*H14
      PFS12=A4*H13
C
C
      A5=0.5*SQRT(1/H9)
      A6=SQRT(1/H)
      A7=0.5*((H)**(-1.5))
C
      PSAFSA=A5*((A6*(2.*C(1,1)*C(1,1)/3.)) - (A7*H11*H11))
      PSBFSA=A5*((A6*((C(1,2)**2.)-(C(1,1)*C(2,2)/3.))) - (A7*H11*H12))
      PSDFSA=A5*((A6*(2.*C(1,1)*C(1,2)/3.)) - (A7*H11*H13))
      PSBFSB=A5*((A6*(2.*(C(2,2)**2.)/3.)) -(A7*H12*H12))
      PSDFSB=A5*((A6*(2.*C(2,2)*C(2,1)/3.)) - (A7*H12*H13))
      PSDFSD=A5*((A6*((C(1,1)*C(2,2))-(C(1,2)**2./3.))) - (A7*H13*H13))
      PSCFSA=A5*((A6*(-C(1,1)*C(3,3)/3.)) - (A7*H11*H14))
      PSCFSB=A5*((A6*(-C(2,2)*C(3,3)/3.)) - (A7*H12*H14))
      PSCFSD=A5*((A6*(-C(3,3)*C(1,2)/3.)) - (A7*H13*H14))
      PSCFSC=A5*((A6*(2.*C(3,3)**2./3.)) - (A7*H14*H14))
C
      RETURN
      END
C
C *****************************************************************
C
      SUBROUTINE PEDWTS
C
C
C THIS SUBROUTINE FINDS THE ELEMENTS OF THE MATRIX [ L ],
C WHICH CONTAINS THE PARTIAL DERAVITIVE OF THE
```

```
C VISCOPLASTIC STRAIN RATE W.R.T. THE STRESS (LAGRANGIAN COORDINATE).
C
C PEDASA= PARTIAL DERIVATIVE OF E DOT 11 W.R.T. S11
C PEDASB= PARTIAL DERIVATIVE OF E DOT 11 W.R.T. S22
C PEDASC= PARTIAL DERIVATIVE OF E DOT 11 W.R.T. S33
C PEDASD= PARTIAL DERIVATIVE OF E DOT 11 W.R.T. S12
C
C -----------------------------------------------------------------
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      COMMON/M1/AJAC,AK,C0,AKAPPA
      COMMON/M2/F,H,H9
      COMMON/M6/AMMA,PGAMF
      COMMON/M7/PFS11,PFS22,PFS33,PFS12,PSAFSA,PSBFSA,PSDFSA,
     .          PSBFSB,PSDFSB,PSDFSD,PSCFSA,PSCFSB,PSCFSD,PSCFSC
      COMMON/M8/Q(4,4)
      COMMON/M14/IFLAG1
C
      PEDASA=AJAC*((PGAMF*PFS11*F*PFS11)+(AMMA*PFS11*PFS11)+
     .(AMMA*F*PSAFSA))
      PEDASB=AJAC*((PGAMF*PFS22*F*PFS11)+(AMMA*PFS22*PFS11)+
     .(AMMA*F*PSBFSA))
      PEDASD=AJAC*((PGAMF*PFS12*F*PFS11)+(AMMA*PFS12*PFS11)+
     .(AMMA*F*PSDFSA))
      PEDBSB=AJAC*((PGAMF*PFS22*F*PFS22)+(AMMA*PFS22*PFS22)+
     .(AMMA*F*PSBFSB))
      PEDBSD=AJAC*((PGAMF*PFS12*F*PFS22)+(AMMA*PFS12*PFS22)+
     .(AMMA*F*PSDFSB))
      PEDDSD=AJAC*((PGAMF*PFS12*F*PFS12)+(AMMA*PFS12*PFS12)+
     .(AMMA*F*PSDFSD))
      PEDBSA=PEDASB
      PEDDSA=PEDASD
      PEDDSB=PEDBSD
C
C     THE FOLLOWING IS USED ONLY FOR AXISYMMETRIC CASE
C
      IF(IFLAG1.NE.3) GO TO 100
      PEDASC=AJAC*((PGAMF*PFS33*F*PFS11)+(AMMA*PFS33*PFS11)+
     .(AMMA*F*PSCFSA))
      PEDBSC=AJAC*((PGAMF*PFS33*F*PFS22)+(AMMA*PFS33*PFS22)+
     .(AMMA*F*PSCFSB))
      PEDDSC=AJAC*((PGAMF*PFS33*F*PFS12)+(AMMA*PFS33*PFS12)+
     .(AMMA*F*PSCFSD))
      PEDCSC=AJAC*((PGAMF*PFS33*F*PFS33)+(AMMA*PFS33*PFS33)+
     .(AMMA*F*PSCFSC))
      PEDCSA=PEDASC
      PEDCSB=PEDBSC
      PEDCSD=PEDDSC
  100 CONTINUE
C
C     BUILD THE ( Q ) MATRIX FOR PLANE STRESS AND PLANE STRAIN
```

```
C
      Q(1,1)=PEDASA
      Q(1,2)=PEDASB
      Q(1,3)=PEDASD
      Q(2,1)=Q(1,2)
      Q(2,2)=PEDBSB
      Q(2,3)=PEDBSD
      Q(3,1)=Q(1,3)
      Q(3,2)=Q(2,3)
      Q(3,3)=PEDDSD
C
C     THIS IS USED FOR AXISYMMETRIC CASE ONLY
C
      IF(IFLAG1.NE.3) GO TO 101
      Q(1,4)=PEDASC
      Q(2,4)=PEDBSC
      Q(3,4)=PEDDSC
      Q(4,4)=PEDCSC
      Q(4,1)=Q(1,4)
      Q(4,2)=Q(2,4)
      Q(4,3)=Q(3,4)
  101 CONTINUE
C
C
      RETURN
      END
C
C ********************************************************************
C
      SUBROUTINE DEVP
C
C THIS PROGRAM EVALUATES THE  ( D ) MATRIX (ELASTIC-VISCOPLASTIC)
C
C COMP(I,J)= COMPLIANCE MATRIX
C DELT.....= TIME INCREMENT ( T(N+1) - T(N) )
C THETA....= CONSTANT  .....=0   FULLY EXPLICIT SCHEME
C                      .....=1   FULLY IMPLICIT SCHEME
C                      .....=0.5 IMPLICIT TRAPEZOIDAL SCHEME
C AMU......= POISSON RATIO
C E........= MODULUS OF ELASTICITY
C D(I,J)...= COMP(I,J) + DELT*THETA*Q(I,J)
C
C --------------------------------------------------------------------
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      COMMON/M8/Q(4,4)
      COMMON/M9/COMP(4,4),AMU,Y,DELT,THETA,G,ALMDA
      COMMON/M10/D(4,4)
      COMMON/M13/D1(4,4)
      COMMON/M14/IFLAG1
C
```

```
C     COMPLIAMCE MATRIX FOR PLANE STRAIN
C
      IF(IFLAG1.NE.1) GO TO 100
      COMP(1,1)= (1.-AMU**2.)/Y
      COMP(1,2)= (-AMU*(1.+AMU))/Y
      COMP(1,3)= 0.0
      COMP(2,1)=COMP(1,2)
      COMP(2,2)=COMP(1,1)
      COMP(2,3)=0.0
      COMP(3,1)=COMP(1,3)
      COMP(3,2)=COMP(2,3)
      COMP(3,3)=1./(2.*G)
  100 CONTINUE
C
C     COMPLIANCE MATRIX FOR PLANE STRESS
C
      IF(IFLAG1.NE.2) GO TO 101
      COMP(1,1)= 1./Y
      COMP(1,2)= -AMU/Y
      COMP(1,3)= 0.0
      COMP(2,1)=COMP(1,2)
      COMP(2,2)=COMP(1,1)
      COMP(2,3)=0.0
      COMP(3,1)=COMP(1,3)
      COMP(3,2)=COMP(2,3)
      COMP(3,3)=1./(2.*G)
  101 CONTINUE
C
C     COMPLIANCE MATRIX FOR AXISYMMETRIC CASE ONLY
C
      IF(IFLAG1.NE.3) GO TO 102
      COMP(1,1)= 1./Y
      COMP(1,2)= -AMU/Y
      COMP(1,3)=0.0
      COMP(1,4)=COMP(1,2)
      COMP(2,1)=COMP(1,2)
      COMP(2,2)=COMP(1,1)
      COMP(2,3)=0.0
      COMP(2,4)=COMP(1,2)
      COMP(3,1)=COMP(1,3)
      COMP(3,2)=COMP(2,3)
      COMP(3,3)=1./(2.*G)
      COMP(3,4)=0.0
      COMP(4,1)=COMP(1,4)
      COMP(4,2)=COMP(2,4)
      COMP(4,3)=COMP(3,4)
      COMP(4,4)=COMP(1,1)
  102 CONTINUE
C
C     FORM THE [ D ]  MATRIX
C
      DO 103 I=1,3
```

```
      DO 103 J=1,3
      D(I,J)=COMP(I,J)+(DELT*THETA*Q(I,J))
  103 CONTINUE
C
C     THE FOLLOWING IS USED FOR AXISYMMETRIC CASE ONLY
C
      IF(IFLAG1.NE.3) GO TO 105
      DO 104 K=1,4
      D(K,4)=COMP(K,4)+(DELT*THETA*Q(K,4))
  104 CONTINUE
      D(4,1)=D(1,4)
      D(4,2)=D(2,4)
      D(4,3)=D(3,4)
  105 CONTINUE
C
C     STORE ( D )  IN  ( D1 )
C
      DO 106 I =1,4
      DO 106 J =1,4
      D1(I,J)=D(I,J)
  106 CONTINUE
C
      RETURN
      END
C
C *******************************************************************
      SUBROUTINE ENVERS
C
C THIS SUBROUTINE EVALUATES THE INVERSE OF THE [ D ] MATRIX USING
C THE GAUSS-JORDAN REDUCTION WITH THE MAXIMUM PIVOT STRATEGY.
C
C     N ......= NO. OF ROWS IN [ A ], WHOSE INVERSE IS DESIRED
C     INDIC ..= COMPUTATIONAL SWITCH
C     EPS ....= MINIMUM ALLOWABLE MAGNITUDE. EPS, FOR A PIVOT ELEMENT
C     DETER ..= DETERMINANT OF THE ORIGINAL COEFFICIENT MATRIX
C
C -------------------------------------------------------------------
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      COMMON/M13/D1(4,4)
      COMMON/M15/B(4,4)
C
      DIMENSION X(4),A(4,4)
C
      N=4
      INDIC=-1
      EPS=1.E-20
C
C     PUT (D1) IN (A)
C
      DO 700 I =1,4
```

```
      DO 700 J =1,4
      A(I,J)=D1(I,J)
700   CONTINUE
      MAX=N
      IF (INDIC.GE.0) MAX=N+1
C
C     ..... CALL SMULA .....
C
      CALL SMULA(N,A,X,EPS,INDIC,11,DETER)
      IF (INDIC.GE.0) GO TO 8
C
C     PUT (A) BACK IN (B)
C
      DO 900 I=1,N
      DO 900 J=1,N
  900 B(I,J)=A(I,J)
C
      GO TO 1
    8 WRITE(6,203) DETER,N
C   8 WRITE(6,203) DETER,N,(X(I),I=1,N)
      IF (INDIC.NE.0) GO TO 1
      WRITE(6,204)
      GO TO 1
C
C     ..... FORMAT FOR INPUT OUTPUT STATEMENTS .....
C
C 100 FORMAT(2I5,E13.7)
C 101 FORMAT(5E13.7)
  200 FORMAT(9H1N      = , I4,/, 9H INDIC = , I4,/,10H EPS =     ,E13.7)
  201 FORMAT(1H ,7E13.7)
  202 FORMAT(10H DETER   = , E13.7,/,22H THE INVERSE MATRIX IS,/,1H )
  203 FORMAT(10H DETER   = , E13.7,/,I2)
C 203 FORMAT(10H DETER   = , E13.7,/,24H THE SOLUTIONS X(1)...X(, I2,
C    1   5H) ARE,/, 1H,/,(1H ,7E13.7))
  204 FORMAT(23H THE INVERSE MATRIX IS ,/, 1H )
    1 CONTINUE
C
      RETURN
      END
C
      SUBROUTINE SMULA(N,A,X,EPS,INDIC,NRC,DETER)
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 A,X,EPS,SIMUL,PIVOT
      DIMENSION IROW(4),JCOL(4),JORD(4),Y(4),A(4,4),X(N)
C
      MAX=N
      IF (INDIC.GE.0) MAX=N+1
C
C     ..... IS N LARGER THAN THE DIMENSIONED VALUE .....
C
      IF (N.LE.10) GO TO 5
```

```
      WRITE(6,200)
      SIMUL=0.
      RETURN
C
C     ..... BEGIN ELIMINATION PROCEDURE .....
C
    5 DETER=1.0
      DO 18 K=1,N
      KM1=K-1
C
C     ..... SEARCH FOR THE PIVOT ELEMENT .....
C
      PIVOT=0.
      DO 11 I=1,N
      DO 11 J=1,N
C
C     ..... SEARCH IROW  JCOL ARRAYS FOR INVALID PIVOT SUBSCRIPTS .....
      IF (K.EQ.1) GO TO 9
      DO 8 ISCAN=1,KM1
      DO 8 JSCAN=1,KM1
      IF (I.EQ.IROW(ISCAN)) GO TO 11
      IF (J.EQ.JCOL(JSCAN)) GO TO 11
    8 CONTINUE
    9 IF (DABS(A(I,J)).LE.DABS(PIVOT)) GO TO 11
      PIVOT =A(I,J)
      IROW(K)=I
      JCOL(K)=J
   11 CONTINUE
C
C     ..... INSURE THAT SELECTED PIVOT IS LARGER THAN EPS .....
C
      IF (DABS(PIVOT).GT.EPS) GO TO 13
      WRITE(6,201)
      SIMUL=0.
      RETURN
C
C     ..... UPDATE THE DETERMINAT VALUE .....
C
   13 IROWK=IROW(K)
      JCOLK=JCOL(K)
      DETER=DETER*PIVOT
C
C     ..... NORMALIZE PIVOT ROW ELEMENTS .....
C
      DO 14 J=1,MAX
   14 A(IROWK,J)=A(IROWK,J)/PIVOT
C
C     ..... CARRY OUT ELIMINATION AND DEVELOP INVERSE .....
C
      A(IROWK,JCOLK)=1./PIVOT
      DO 18 I=1,N
      AIJCK = A(I,JCOLK)
```

```
      IF (I.EQ.IROWK) GO TO 18
      A(I,JCOLK) = - AIJCK/PIVOT
      DO 17 J=1,MAX
   17 IF (J.NE.JCOLK) A(I,J)=A(I,J) - AIJCK*A(IROWK,J)
   18 CONTINUE
C
C     ..... ORDER SOLUTION VALUES (IF ANY) AND CREATE JORD ARRAY .....
C
      DO 20 I=1,N
      IROWI=IROW(I)
      JCOLI=JCOL(I)
      JORD(IROWI)=JCOLI
   20 IF (INDIC.GE.0) X(JCOLI)=A(IROWI,MAX)
C
C     ..... ADJUST SIGN OF DETERMINANT .....
C
      INTCH=0
      NM1=N-1
      DO 22 I=1,NM1
      IP1=I+1
      DO 22 J=IP1,N
      IF (JORD(J).GE.JORD(I)) GO TO 22
      JTEMP=JORD(J)
      JORD(J)=JORD(I)
      JORD(I)=JTEMP
      INTCH=INTCH+1
   22 CONTINUE
      IF (INTCH/2*2.NE.INTCH ) DETER = - DETER
C
C     ..... IF INDIC IS POSITIVE RETURN WITH RESULTS .....
C
      IF (INDIC.LE.0) GO TO 26
      SIMUL=DETER
      RETURN
C
C     ..... IF INDIC IS NEGATIVE OR ZERO, UNSCRAMBLE THE INVERSE
C           FIRST BY ROWS .....
   26 DO 28 J=1,N
      DO 27 I=1,N
      IROWI=IROW(I)
      JCOLI=JCOL(I)
   27 Y(JCOLI)=A(IROWI,J)
      DO 28 I=1,N
   28 A(I,J)=Y(I)
C     ..... THEN BY COLUMNS .....
      DO 30 I=1,N
      DO 29 J=1,N
      IROWJ=IROW(J)
      JCOLJ=JCOL(J)
   29 Y(IROWJ)=A(I,JCOLJ)
      DO 30 J=1,N
   30 A(I,J)=Y(J)
```

```
C
C     ..... RETURN FOR INDIC NEGATIVE OR ZERO .....
C
      SIMUL=DETER
      RETURN
C
C     ..... FORMAT FOR OUTPUT STATEMENT .....
C
  200 FORMAT( 40H N IS GREATER THAN THE DIMENSIONED VALUE)
  201 FORMAT( 30H THE PIVOT IS SMALLER THAN EPS)
C
      END
C
C     ***************************************************************
C
      SUBROUTINE SOLVE
C
C THIS SUBROUTINE SOLVES INCREMENTALSTRESS(DELS11,DELS22,DELS33,DELS12)
C AND CALCULATES THE TOTAL STRESSES S11,S22,S33,S12
C DELS(I,J).....=STRESS INCREMENT OF COMPONENT I,J
C DELET(I,J)....=TOTAL STRAIN INCREMENT OF COMPONENT I,J
C DELEVP(I,J)...=VISCO PLASTIC STRAIN INCREMENT OF COMPONENT I,J
C DELT..........=TIME INCREMENT
C
C     ---------------------------------------------------------------
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      COMMON/M0/S(3,3),C(3,3),A(3,3),E(3,3)
      COMMON/M12/DELEE(3,3),DELEVP(3,3),DELET(3,3),DELA(3,3),DELS(3,3)
      COMMON/M13/D1(4,4)
      COMMON/M14/IFLAG1
      COMMON/M15/B(4,4)
C
      DIMENSION DELES(3,3)
C
C     INITIALIZE DELES(I,J)
C
      DO 20 I=1,3
      DO 20 J=1,3
      DELES(I,J)=0.0
      DELS(I,J)=0.0
   20 CONTINUE
      DELT=1.0
C
C     COMPUTE THE VECTOR DELES(I,J)= DELET(I,J) - DELT*DELEVP(I,J)
C
      DO 21 I=1,3
      DELES(I,I)=DELET(I,I)-(DELT*DELEVP(I,I))
   21 CONTINUE
      DELES(1,2)=DELET(1,2)-(DELT*DELEVP(1,2))
C
```

```
C     CALCULATES STRESS INCREMENT FOR THE PLANE STRESS AND PLANE STRAIN
C
      DELS(1,1)= B(1,1)*DELES(1,1) + B(1,2)*DELES(2,2)
     .         + B(1,3)*DELES(1,2) + B(1,4)*DELES(3,3)
      DELS(2,2)= B(2,1)*DELES(1,1) + B(2,2)*DELES(2,2)
     .         + B(2,3)*DELES(1,2) + B(2,4)*DELES(3,3)
      DELS(1,2)= B(3,1)*DELES(1,1) + B(3,2)*DELES(2,2)
     .         + B(3,3)*DELES(1,2) + B(3,4)*DELES(3,3)
C
C     CALCULATE THE TOTAL STRESS
C
      S(1,1)=S(1,1)+DELS(1,1)
      S(2,2)=S(2,2)+DELS(2,2)
      S(1,2)=S(1,2)+DELS(1,2)
C
C     THIS IS USED FOR AXISYMMETRIC CASE ONLY
C
      IFLAG1=3
      IF(IFLAG1.NE.3) GOTO 10
      DELS(3,3)= B(4,1)*DELES(1,1) + B(4,2)*DELES(2,2)
     .         + B(4,3)*DELES(1,2) + B(4,4)*DELES(3,3)
C
      S(3,3)=S(3,3)+DELS(3,3)
   10 CONTINUE
C
      RETURN
      END
```

Appendix B

MAIN PROGRAM AND SUBROUTINES

The main program of SAVPA as well as the subroutines that are controlled by the main program are briefly discussed and listed below.

1. Main Program

The main program increments the material strains and controls all the subroutines to compute the second Piola-Kirchhoff stress increments, the material strain tensor, and the second Piola-Kirchoff stress tensor. The main program also computes the increments of the stress shift ΔA_{AB}, the plastic work done κ, and the stress shift tensor A_{AB}.

2. Subroutine INPUT

This subroutine contains most of the constants used throughout the program. It allows the user to set the values of those constants according to what is required of the program to solve for. As a reminder, the choices that govern the numerical solutions are listed at the beginning of Chapter 4. Moreover, subroutine INITIAL initializes the values of the elements of most of the arrays used throughout the program. It follows from the above that subroutine INITIAL acts as an input data file.

3. Subroutine AJACOB

This subroutine computes the determinant of the Jacobian of the deformation, and Green's deformation tensor.

4. Subroutine YIELDF

This subroutine evaluates the yield function expressed in equation (19) in Chapter 2 for two-dimensional problems and reports its value to the main program through F to check if the material has yielded. It is worth noting here that if F is less than zero, the material is undergoing elastic deformation. However, if F is greater than or equal to zero, the material is in a state of visco plastic flow. The yield function expressed in equation (19) is broken down into nine parts when computed inthe subroutine YIELDF. Therefore, we have

$$F = \mathrm{SQRT}\ [(H_1 + H_2 + H_3 + H_4 + H_5 + H_6 + H_7 + H_8)\ \frac{1}{H9}] - 1 \qquad \text{(B-1)}$$

where equation (B-1) appears at the end of subroutine YIELDF and its variables are defined here as follows:

$$\begin{aligned} H1 = &\ \tfrac{1}{3}\, s_{11}^2\, C_{11}^2 + s_{11}\, s_{22}\, C_{12}^2 + \tfrac{2}{3}\, s_{11}\, s_{12}\, C_{11}\, C_{12} \\ &+ \tfrac{1}{3}\, s_{22}^2\, C_{22}^2 + \tfrac{2}{3}\, s_{22}\, s_{12}\, C_{22}\, C_{21} + \tfrac{1}{3}\, s_{33}^2\, C_{33}^2 \end{aligned} \qquad \text{(B-2)}$$

$$\begin{aligned} H2 = &\ \tfrac{1}{2}\, s_{12}^2\, (C_{11}\, C_{22} - \tfrac{1}{3}\, C_{12}^2) - \tfrac{1}{3}\, s_{11}\, s_{22}\, C_{11}\, C_{22} \\ &- \tfrac{1}{3}\, s_{11}\, s_{33}\, C_{11}\, C_{33} - \tfrac{1}{3}\, s_{22}\, s_{33}\, C_{22}\, C_{33} \\ &- \tfrac{1}{3}\, s_{33}\, s_{12}\, C_{33}\, C_{12} \end{aligned} \qquad \text{(B-3)}$$

$$\begin{aligned} H3 = &\ s_{11}\, (- \tfrac{2}{3}\, A_{11}\, C_{11}^2 - A_{22}\, C_{12}^2 - \tfrac{2}{3}\, A_{12}\, C_{11}\, C_{12} \\ &+ \tfrac{1}{3}\, A_{22}\, C_{11}\, C_{22} + \tfrac{1}{3}\, A_{33}\, C_{11}\, C_{33}) \end{aligned} \qquad \text{(B-4)}$$

$$\begin{aligned} H4 = &\ s_{22}\, (- A_{11}\, C_{21}^2 - \tfrac{2}{3}\, A_{22}\, C_{22} - \tfrac{2}{3}\, A_{12}\, C_{21}\, C_{22} \\ &+ \tfrac{1}{3}\, A_{11}\, C_{11}\, C_{22} + \tfrac{1}{3}\, A_{33}\, C_{22}\, C_{33}) \end{aligned} \qquad \text{(B-5)}$$

$$\begin{aligned} H5 = &\ s_{33}\, (- \tfrac{2}{3}\, A_{33}\, C_{33}^2 + \tfrac{1}{3}\, A_{11}\, C_{11}\, C_{33} \\ &+ \tfrac{1}{3}\, A_{22}\, C_{22}\, C_{33} + \tfrac{1}{3}\, A_{12}\, C_{12}\, C_{33}) \end{aligned} \qquad \text{(B-6)}$$

$$H6 = s_{12} \left(- \frac{2}{3} A_{11} C_{11} C_{12} - \frac{2}{3} A_{22} C_{22} C_{12} - A_{12} C_{11} C_{22} + \frac{1}{3} A_{33} C_{12} C_{33} + \frac{1}{3} A_{12} C_{12}^2\right) \quad \text{(B-7)}$$

$$H7 = \frac{1}{3} A_{11}^2 C_{11}^2 + A_{11} A_{22} C_{12}^2 + \frac{2}{3} A_{11} A_{12} C_{11} C_{12} + \frac{1}{3} A_{22}^2 C_{22}^2 + \frac{2}{3} A_{22} A_{12} C_{22} C_{21} + \frac{1}{3} A_{33}^2 C_{33}^2 \quad \text{(B-8)}$$

$$H8 = \frac{1}{2} A_{12} \left(C_{11} C_{22} - \frac{1}{3} C_{12}^2\right) - \frac{1}{3} A_{11} A_{22} C_{11} C_{22} - \frac{1}{3} A_{11} A_{33} C_{11} C_{33} - \frac{1}{3} A_{22} A_{33} C_{22} C_{33} - \frac{1}{3} A_{33} A_{12} C_{33} C_{12} \quad \text{(B-9)}$$

$$H9 = J^2 \left(K^2 + CO\kappa\right) \quad \text{(B-10)}$$

and CO is the isotropic-hardening material parameter c obtained from the formulation in Chapter 3. Note that κ, which is the viscoplastic work done, is computed in the main program. When the von Mises yield criterion is used in the constitutive modeling, which is the case in this work, the constant k is given by

$$K = \frac{s_y}{\sqrt{3}} \quad \text{(B-11)}$$

where s_y is the initial yield stress obtained from the experimental data of the uniaxial tests.

5. Subroutine GAMMA

This subroutine evaluates the material function Gamma (Γ) expressed in equation (93) of Chapter 3.

6. Subroutine PFWRTS

This subroutine evaluates the derivatives of the yield function expressed in equation (B-1) with respect to the stresses s_{11}, s_{22}, s_{33}, and s_{12}. Also, it computes $(\partial/\partial \underset{\sim}{s})(\partial F/\partial \underset{\sim}{s})$ for two-dimensional problems (i.e., s_{11}, s_{22}, s_{33}, and s_{12}).

The derivative of the yield function in B-1 with respect to s_{11}, s_{22}, s_{33}, and s_{12} reduces to

$$\frac{\partial F}{\partial s_{11}} = A4 \cdot H11 \tag{B-12}$$

$$\frac{\partial F}{\partial s_{22}} = A4 \cdot H12 \tag{B-13}$$

$$\frac{\partial F}{\partial s_{33}} = A4 \cdot H14 \tag{B-14}$$

$$\frac{\partial F}{\partial s_{12}} = A4 \cdot H13 \tag{B-15}$$

where

$$\begin{aligned} H11 = [&\tfrac{2}{3} s_{11} C_{11}^2 + s_{22} C_{12}^2 + \tfrac{2}{3} s_{12} C_{11} C_{12} - \tfrac{1}{3} s_{22} C_{11} C_{22} \\ &- \tfrac{1}{3} s_{33} C_{11} C_{33} + (- \tfrac{2}{3} A_{11} C_{11}^2 - A_{22} C_{12}^2 - \tfrac{2}{3} A_{12} C_{11} C_{12} \\ &+ \tfrac{1}{3} A_{22} C_{11} C_{22} + \tfrac{1}{3} A_{33} C_{11} C_{33}) \end{aligned} \tag{B-16}$$

$$\begin{aligned} H12 = &s_{11} C_{12}^2 + \tfrac{2}{3} s_{22} C_{22}^2 + \tfrac{2}{3} s_{12} C_{22} C_{21} - \tfrac{1}{3} s_{11} C_{11} C_{22} \\ &- \tfrac{1}{3} s_{33} C_{22} C_{33} + (- A_{11} C_{21}^2 - \tfrac{2}{3} A_{22} C_{22}^2 - \tfrac{2}{3} A_{12} C_{21} C_{22} \\ &+ \tfrac{1}{3} A_{11} C_{22} C_{11} + \tfrac{1}{3} A_{33} C_{22} C_{33}) \end{aligned} \tag{B-17}$$

$$\mathrm{H13} = \frac{2}{3} s_{11}\, C_{11}\, C_{12} + \frac{2}{3} s_{22}\, C_{22}\, C_{21} + s_{12}\, (C_{11}\, C_{22} - \frac{1}{3} C_{12}^2)$$

$$- \frac{1}{3} s_{33}\, C_{33}\, C_{12} + (- \frac{2}{3} A_{11}\, C_{11}\, C_{21} - \frac{2}{3} A_{22}\, C_{12}\, C_{22}$$

$$- A_{12}\, C_{11}\, C_{22} + \frac{1}{3} A_{33}\, C_{12}\, C_{33} + \frac{1}{3} A_{12}\, C_{12}^2] \qquad \text{(B-18)}$$

$$\mathrm{H14} = \frac{2}{3} s_{33}\, C_{33}^2 - \frac{1}{3} s_{11}\, C_{11}\, C_{33} - \frac{1}{3} s_{22}\, C_{22}\, C_{33}$$

$$- \frac{1}{3} s_{12}\, C_{33}\, C_{12} + (- \frac{2}{3} A_{33}\, C_{33}^2 + \frac{1}{3} A_{11}\, C_{33}\, C_{11}$$

$$+ \frac{1}{3} A_{22}\, C_{33}\, C_{22} + \frac{1}{3} A_{12}\, C_{33}\, C_{12})] \qquad \text{(B-19)}$$

$$\mathrm{A4} = \frac{1}{2}\ \mathrm{SQRT}\ (\frac{1}{J^2(K^2 + \mathrm{CO}\kappa)})\ (\mathrm{H})^{-1/2} \qquad \text{(B-20)}$$

The derivative with respect to the stress tensor (s_{11}, s_{22}, s_{33}, and s_{12}) of the yield function F with respect to the stress tensor $\underset{\sim}{s}$ $((\partial/\partial \underset{\sim}{s})(\partial F/\partial \underset{\sim}{s}))$ yields:

$$\frac{\partial}{\partial s_{11}} (\frac{\partial F}{\partial s_{11}}) = \mathrm{A5}\ [(\mathrm{A6}\ (\frac{2}{3} C_{11}C_{11})) - (\mathrm{A7 \cdot H11 \cdot H11})] \qquad \text{(B-21)}$$

$$\frac{\partial}{\partial s_{22}} (\frac{\partial F}{\partial s_{11}}) = \mathrm{A5}\ [(\mathrm{A6}\ (C_{12}C_{12} - \frac{1}{3} C_{11}C_{22})) - (\mathrm{A7 \cdot H11 \cdot H12})] \qquad \text{(B-22)}$$

$$\frac{\partial}{\partial s_{12}} (\frac{\partial F}{\partial s_{11}}) = \mathrm{A5}\ [(\mathrm{A6}\ (\frac{2}{3} C_{11}C_{12})) - (\mathrm{A7 \cdot H11 \cdot H13})] \qquad \text{(B-23)}$$

$$\frac{\partial}{\partial s_{22}} (\frac{\partial F}{\partial s_{22}}) = \mathrm{A5}\ [(\mathrm{A6}\ (\frac{2}{3} C_{22}C_{22})) - (\mathrm{A7 \cdot H11 \cdot H13})] \qquad \text{(B-24)}$$

$$\frac{\partial}{\partial s_{12}} (\frac{\partial F}{\partial s_{22}}) = \mathrm{A5}\ [(\mathrm{A6}\ (\frac{2}{3} C_{12}C_{22})) - (\mathrm{A7 \cdot H11 \cdot H12})] \qquad \text{(B-25)}$$

$$\frac{\partial}{\partial s_{12}} (\frac{\partial F}{\partial s_{12}}) = \mathrm{A5}\ [(\mathrm{A6}\ (C_{11}C_{22} - \frac{1}{3} C_{12}C_{12})) - (\mathrm{A7 \cdot H13 \cdot H13})] \qquad \text{(B-26)}$$

$$\frac{\partial}{\partial s_{33}} (\frac{\partial F}{\partial s_{11}}) = \mathrm{A5}\ [(\mathrm{A6}\ (\frac{1}{3} C_{11}C_{33})) - (\mathrm{A7 \cdot H11 \cdot H14})] \qquad \text{(B-27)}$$

$$\frac{\partial}{\partial s_{33}} (\frac{\partial F}{\partial s_{22}}) = \mathrm{A5}\ [(\mathrm{A6}\ (- \frac{1}{3} C_{22}C_{33})) - (\mathrm{A7 \cdot H12 \cdot H14})] \qquad \text{(B-28)}$$

$$\frac{\partial}{\partial s_{33}} \left(\frac{\partial F}{\partial s_{12}}\right) = A5 \; [(A6 \; (- \frac{1}{3} \; C_{33}C_{12})) - (A7 \cdot H13 \cdot H14)] \tag{B-29}$$

$$\frac{\partial}{\partial s_{33}} \left(\frac{\partial F}{\partial s_{33}}\right) = A5 \; [(A6 \; (\frac{2}{3} \; C_{33}C_{33})) - (A7 \cdot H14 \cdot H14)] \tag{B-30}$$

where

$$A5 = \frac{1}{2} \; SQRT \; \left(\frac{1}{H9}\right) \tag{B-31}$$

$$A6 = (H)^{-1/2} \tag{B-32}$$

$$A7 = \frac{1}{2} \; (H)^{-3/2} \tag{B-33}$$

H is equal to the sum of H1 through H8 as defined by equations (B-2) to (B-9).

7. Subroutine PEDWTS

This subroutine determines the derivative of the material visco-plastic strain rate with respect to the material stress (see equation (41) in Chapter 2). Subroutine PEDWTS computes the matrix Q whose elements are $\partial \dot{\tilde{e}}''/\partial \tilde{s}$. The following discussion explains the theoretical formulation of $\partial \dot{\tilde{e}}''/\partial \tilde{s}$. From equation (20) in Chapter 2, the visco-plastic strain rate is a function of:

$$\dot{\tilde{e}}'' = \dot{\tilde{e}}'' \; (J, \; \gamma, \; F, \; \frac{\partial F}{\partial \tilde{s}}) \tag{B-34}$$

Therefore, the derivative of $\dot{\tilde{e}}''$ with respect to $\tilde{s}$ can be evaluated as:

$$\frac{\partial \dot{e}''_{11}}{\partial s_{11}} = J \; [\frac{\partial \gamma}{\partial F} \frac{\partial F}{\partial s_{11}} \; F \; \frac{\partial F}{\partial s_{11}} + \gamma \; \frac{\partial F}{\partial s_{11}} \; \frac{\partial F}{\partial s_{11}} + \gamma \; F \; \frac{\partial}{\partial s_{11}} \; (\frac{\partial F}{\partial s_{11}})] \tag{B-35}$$

$$\frac{\partial \dot{e}''_{11}}{\partial s_{22}} = J \; [\frac{\partial \gamma}{\partial F} \frac{\partial F}{\partial s_{22}} \; F \; \frac{\partial F}{\partial s_{11}} + \gamma \; \frac{\partial F}{\partial s_{22}} \; \frac{\partial F}{\partial s_{11}} + \gamma \; F \; \frac{\partial}{\partial s_{22}} \; (\frac{\partial F}{\partial s_{11}})] \tag{B-36}$$

$$\frac{\partial \dot{e}''_{11}}{\partial s_{12}} = J \; [\frac{\partial \gamma}{\partial F} \frac{\partial F}{\partial s_{12}} \; F \; \frac{\partial F}{\partial s_{11}} + \gamma \; \frac{\partial F}{\partial s_{12}} \; \frac{\partial F}{\partial s_{11}} + \gamma \; F \; \frac{\partial}{\partial s_{12}} \; (\frac{\partial F}{\partial s_{11}})] \tag{B-37}$$

$$\frac{\partial \dot{e}''_{22}}{\partial s_{11}} = J \; [\frac{\partial \gamma}{\partial F} \frac{\partial F}{\partial s_{11}} \; F \; \frac{\partial F}{\partial s_{22}} + \gamma \; \frac{\partial F}{\partial s_{11}} \; \frac{\partial F}{\partial s_{22}} + \gamma \; F \; \frac{\partial}{\partial s_{11}} \; (\frac{\partial F}{\partial s_{22}})] \tag{B-38}$$

$$\frac{\partial \dot{e}''_{22}}{\partial s_{22}} = J\ [\frac{\partial \gamma}{\partial F} \frac{\partial F}{\partial s_{22}}\ F\ \frac{\partial F}{\partial s_{22}} + \gamma \frac{\partial F}{\partial s_{22}} \frac{\partial F}{\partial s_{22}} + \gamma\ F\ \frac{\partial}{\partial s_{22}} (\frac{\partial F}{\partial s_{22}})] \quad \text{(B-39)}$$

$$\frac{\partial \dot{e}''_{22}}{\partial s_{12}} = J\ [\frac{\partial \gamma}{\partial F} \frac{\partial F}{\partial s_{12}}\ F\ \frac{\partial F}{\partial s_{22}} + \gamma \frac{\partial F}{\partial s_{12}} \frac{\partial F}{\partial s_{22}} + \gamma\ F\ \frac{\partial}{\partial s_{12}} (\frac{\partial F}{\partial s_{22}})] \quad \text{(B-40)}$$

$$\frac{\partial \dot{e}''_{12}}{\partial s_{12}} = J\ [\frac{\partial \gamma}{\partial F} \frac{\partial F}{\partial s_{12}}\ F\ \frac{\partial F}{\partial s_{12}} + \gamma \frac{\partial F}{\partial s_{12}} \frac{\partial F}{\partial s_{12}} + \gamma\ F\ \frac{\partial}{\partial s_{12}} (\frac{\partial F}{\partial s_{12}})] \quad \text{(B-41)}$$

$$\frac{\partial \dot{e}''_{11}}{\partial s_{33}} = J\ [\frac{\partial \gamma}{\partial F} \frac{\partial F}{\partial s_{33}}\ F\ \frac{\partial F}{\partial s_{11}} + \gamma \frac{\partial F}{\partial s_{33}} \frac{\partial F}{\partial s_{11}} + \gamma\ F\ \frac{\partial}{\partial s_{33}} (\frac{\partial F}{\partial s_{11}})] \quad \text{(B-42)}$$

$$\frac{\partial \dot{e}''_{22}}{\partial s_{33}} = J\ [\frac{\partial \gamma}{\partial F} \frac{\partial F}{\partial s_{33}}\ F\ \frac{\partial F}{\partial s_{22}} + \gamma \frac{\partial F}{\partial s_{33}} \frac{\partial F}{\partial s_{22}} + \gamma\ F\ \frac{\partial}{\partial s_{33}} (\frac{\partial F}{\partial s_{22}})] \quad \text{(B-43)}$$

$$\frac{\partial \dot{e}''_{12}}{\partial s_{33}} = J\ [\frac{\partial \gamma}{\partial F} \frac{\partial F}{\partial s_{33}}\ F\ \frac{\partial F}{\partial s_{12}} + \gamma \frac{\partial F}{\partial s_{33}} \frac{\partial F}{\partial s_{12}} + \gamma\ F\ \frac{\partial}{\partial s_{33}} (\frac{\partial F}{\partial s_{12}})] \quad \text{(B-44)}$$

$$\frac{\partial \dot{e}''_{33}}{\partial s_{33}} = J\ [\frac{\partial \gamma}{\partial F} \frac{\partial F}{\partial s_{33}}\ F\ \frac{\partial F}{\partial s_{33}} + \gamma \frac{\partial F}{\partial s_{33}} \frac{\partial F}{\partial s_{33}} + \gamma\ F\ \frac{\partial}{\partial s_{33}} (\frac{\partial F}{\partial s_{33}})] \quad \text{(B-45)}$$

8. Subroutine DEVP

This subroutine determines the elasto/viscoplastic stiffness tensor. The constant θ is set to 0.5 for implicit trapezoidal scheme which is generally known as the Crank-Nicolson rule used in linear equations. Also, the time increment was set to 1.0.

9. Subroutine INVERSE

This subroutine computes the inverse of the matrix determined in subroutine DEVP. Subroutine INVERSE uses the maximum pivot strategy to compute the inverse.

10. Subroutine SOLVE

This subroutine evaluates the second Piola-Kirchhoff stress increments and the total stresses.

Appendix C

```
10 '*********************************************************************
20 '*                                                                   *
30 '*     THIS PROGRAM CONTROLS THE MTS TESTING SYSTEM WITH THE DASH16  *
40 '*                                                                   *
50 '*     MERTABYTE DATA ACQUISITION BOARD.                             *
60 '*                                                                   *
70 '*     PROGRAM CAPABILITIES --- UNIAXIAL LOADING-UNLOADING FOR       *
80 '*                              CONSTANT SPATIAL STRAIN RATE, CREEP  *
85 '*                              AND RELAXATION.                      *
90 '*                                                                   *
100 '*    DEVELOPED BY: LOUAY N. MOHAMMAD                               *
110 '*    SUPERVISOR  : DR. GEORGE Z. VOYIADJIS                         *
120 '*    DEVELOPED AT: LOUISIANA STATE UNIVERSITY                      *
140 '*                                                                  *
150 '*    LOAD-UNLOAD CAPABILITY (ENTER 8 FOR LOADING                   *
160 '*                            ENTER 2 FOR UNLOADING                 *
170 '*                            ENTER 7 FOR HOLD  )                   *
180 '*                                                                  *
190 '*    CHANNELS USED --------- 0 ......... LOAD CELL                 *
200 '*                            1 ......... EXTENSOMETER              *
210 '*                            2 ......... STRAIN GAGE               *
220 '*                            3 ......... ACTUATOR                  *
230 '*                                                                  *
240 '*  ARRAYS USED ------------- LC(J)...... LOAD IN POUNDS            *
250 '*                            EX(J)...... TOTAL ELONGATION IN INCHES *
260 '*                            SAG(J)..... STRAIN GAGE OUTPUT IN     *
270 '*                                        COUNTS                    *
280 '*                            SG(J)...... STRAIN GAGE OUTPUT IN     *
290 '*                                        MICRO-STRAIN              *
300 '*                                                                  *
310 '********************************************************************
320 '
330 '
340 ' DEFINE WORK SPACE & INITIALIZE THE BOARD
350 '
360 '
370 CLEAR, 49152!
380 SCREEN 0,0,0:KEY OFF:CLS
390 DEF SEG = 0
400 SG = 256 * PEEK(&H511) + PEEK(&H510)
410 SG = SG + 49152!/16
420 DEF SEG = SG
430 BLOAD "AIO16.BIN", 0
440 DIM DIO%(8)
450 DIO%(0)=768 : DIO%(1) = 2 : DIO%(2) = 3
460 DASH = 0
470 FLAG% = 0
480 MD% = 0
```

```
490 CALL AIO16 (MD%, DIO%(0),FLAG%)
500 '
510 DIM LC(1500),EX(1500),SAG(1500),SG(1500)
520 '
530 LOCATE 25,3 :PRINT " .....Initializing --- Please Wait " :LOCATE 25,3
540 '
550 ' INITIALIZE THE ARRAYS
560 FOR I=0 TO 1500
570 LC(I)=0!
580 EX(I)=0!
590 SAG(I)=0!
600 SG(I)=0!
610 NEXT
620 '
630 '*************  CALIBRATION FACTORS  ******************************
640 '
650 CLS:SCREEN 1:COLOR 8,0: LINE (0,0)-(319,199),1,B
660 LOCATE 2,13:PRINT"TEST INFORMATION"
670 LOCATE 9,2: INPUT"NO. OF CONVERSIONS/SEC ( > 0 )";NOC
680 IF NOC < = 0 THEN 650
690 CO=1.0035:C1=1.010101 :C3=1
700 NO=(50000! * CO )/2047 :' LOAD FACTOR
710 N1=(.4895 * C1 )/2029 :'EXTENSOMETER FACTOR (RANGE=1)---- A/D
720 NC= .2460227/4095 :'     EXTENSOMETER FACTOR (RANGE=1)---- D/A
730 N3=(2.2 * C3 )/902 :' LVDT FACTOR
740 ZO=NO/NOC :Z1=N1/NOC :Z3=N3/NOC
750 '
760 '****************** TEST INFORMATION **********************
770 '
780 CLS:SCREEN 1:COLOR 8,0: LINE (0,0)-(319,199),1,B
790 LOCATE 2,13:PRINT"TEST INFORMATION"
800 LOCATE 5,2: INPUT"TEST ID                    ";ID$
810 LOCATE 8,2: INPUT"DATE OF TEST               ";DA$
820 LOCATE 11,2: INPUT"TYPE OF SPECIMEN          ";TY$
830 LOCATE 14,2: INPUT"STRAIN RATE (LOADING)     ";SR1
840 LOCATE 17,2: INPUT"STRAIN RATE (UNLOADING) ";SR2
850 LOCATE 20,2: INPUT"GAGE LENGTH               ";IL
860 LOCATE 23,2: INPUT"DIAMETER                  ";ID
870 '
880 RFO=1! :' COM IS THE % OF THE SPAN USED -SPAN SET- (I.E. 0.16=16% OF
COM)
890  MUL1=(SR1*IL*IL)/(NC*RF0):  ' USED IN CALCULATING DISP. INCREMENT
-LOADING-
900  MUU1=(SR2*IL*IL)/(NC*RF0):  ' USED IN CALCULATING DISP. INCREMENT
-UNLOADING-
910 '
920 ' SELECT PROPER CHANNEL FOR CALIBRATION
930 '
940 CLS: LINE (0,0)-(319,199),1,B
950 LOCATE 5,3 :PRINT " CHANNEL 0 = LOAD CELL  "
960 LOCATE 7,3 :PRINT " CHANNEL 1 = EXETENSOMETER "
970 LOCATE 9,3 :PRINT " CHANNEL 2= STRAIN GAGE 1 "
```

```
980 LOCATE 11,3 :PRINT " CHANNEL 3 = LVDT        "
990 LOCATE 15,2 :PRINT "SELECT CHANNEL TO MONITOR (0 - 3) "
1000 A$=INKEY$ :IF A$="" THEN 1000
1010 IF ASC(A$) >47 AND ASC(A$) < 58 THEN ACH=ASC(A$) - 48
1020 CLS:LINE (0,0)-(319,199),1,B
1030 LOCATE 5,5 :PRINT"CHANNEL       = "
1040 LOCATE 7,5 :PRINT"LOAD          = "
1050 LOCATE 9,5 :PRINT"DISP. (EXET) = "
1060 LOCATE 11,5 :PRINT"STRAIN G1     = "
1070 LOCATE 13,5 :PRINT"DISP. (LVDT) = "
1080 LOCATE 20,11 :PRINT "Press ESC To Finish "
1090 LOCATE 22,5  :PRINT"Press S to Select Another Channel "
1100 '
1110 ' SET MULTIPLEXER SCAN LIMIT
1120 '
1130 MD%=1
1140 DIO%(0)=ACH :'LOWER SCAN LIMIT
1150 DIO%(1)=ACH :'UPPER SCAN LIMIT
1160 CALL AIO16 (MD%,DIO%(0),FLAG%)
1170 IF FLAG% <> 0 THEN PRINT " MULTIPLEXER SCAN LIMIT ERROR "
1180 '
1190 ' PERFORM A/D CONVERSION USING MODE 3
1200 '
1210 MD%=3
1220 CALL AIO16 (MD%,DIO%(0),FLAG%)
1230 IF FLAG% <> 0 THEN PRINT " A/D CONVERSION ERROR  "
1240 LOCATE 5,20 :PRINT DIO%(1)
1250 IF DIO%(1)=0 THEN LOCATE 7,20 :PRINT USING "#####";DIO%(0)*C0
1260 IF DIO%(1)=1 THEN LOCATE 9,20 :PRINT USING " ##.####";DIO%(0)*N1
1270 IF DIO%(1)=2 THEN LOCATE 11,20 :PRINT DIO%(0)
1280 IF DIO%(1)=3 THEN LOCATE 13,20 :PRINT DIO%(0)
1290 A$=INKEY$ : IF A$=CHR$(27) THEN 1320
1300 IF A$="S" OR A$="s" THEN 940
1310 GOTO 1210
1320 IF ACH=0 OR ACH=1 THEN 1400
1330 IF ACH=3 THEN 1400
1340 IF ACH=2 THEN  GOSUB 2370
1350 CLS:LINE (0,0)-(319,199),1,B
1360 LOCATE 13,3:PRINT"Calibrate Another Strain Gage <Y,N> "
1370 A$=INKEY$: IF A$="" THEN 1370
1380 IF A$ = "Y" OR A$ = "y" THEN 940
1390 IF A$ <> "N" AND A$ <> "n" THEN 1370
1400 CLS: SCREEN 0 : WIDTH 80
1410 PT=0 : XX%=0 :T$="0" :S$="0":II=0:I=0
1420 '
1430 CLS:PRINT
1440 PRINT "     TEST ID                    : ";ID$
1450 PRINT "     DATE OF TEST               : ";DA$
1460 PRINT "     TYPE OF SPECIMEN           : ";TY$
1470 PRINT USING "     STRAIN RATE (LOADING)    : #.##^^^^";SR1
1480 PRINT USING "     STRAIN RATE (UNLOADING) : #.##^^^^";SR2
1490 PRINT USING "     GAGE LENGTH              : #.###";IL
```

```
1500 PRINT USING "    DIAMETER                    : #.###";ID
1510 PRINT
1520 PRINT"    PT    II    LOAD  EXTEN.    SG1  XX%"
1530 '

1540 '   SET MULTIPEXER TO SCAN CHANNELS 0 - 2
1550 '
1560 MD%=1
1570 DIO%(0)=0
1580 DIO%(1)=2
1590 CALL AIO16 (MD%,DIO%(0),FLAG%)
1600 IF FLAG% <> 0 THEN PRINT " MULTIPLEXER SCAN LIMIT ERROR "
1610 '
1620 ' INITIALIZE (I.E.,ZERO INPUT COMMAND  TO DA#0 )
1630 '
1640 '                USING MODE 15
1650 '
1660 TIME$="0:0:0"
1670 GOSUB 2690 :' APPLY ZERO DISPLACEMENT & READ CHANNELS 0 - 3
1680 LC(II)=A0*Z0
1690 EX(II)=A1*Z1
1700 SAG(II)=A2/NOC
1710 LOCATE 12,1
1720  PRINT  USING"    ####   ####   ######   ##.####
######";PT,II,LC(II),EX(II),SAG(II);
1730 PRINT USING " ####";XX%;
1740 PRINT "    "T$"  "S$
1750 PRINT " HIT ANY KEY TO START "
1760 C$=INKEY$ :IF C$="" THEN 1760
1770 '
1780 '
1790 ' APPLY DISPLACEMENT USING DA#0 ----MODE 15----
1800 '
1810 LOCATE 25,3:PRINT" ENTER        8... LOAD    2... UNLOAD     ESC...
ABORT "
1820 TIME$="0:0:0" :T$="0":S$="0"
1830 '
1840 A$=INKEY$ : IF A$=CHR$(56) THEN VO=0 :'CHR$(56)= 8
1850 IF A$=CHR$(50) THEN VO=1 :'CHR$(50)= 2
1860 IF A$=CHR$(51) THEN VO=2 :'CHR$(51)= 3
1870 IF A$=CHR$(27) THEN 1950
1880 '
1890 PT=PT+1
1900 IF II > 1500 THEN 1960  :' PT MUST NOT EXCEED DIM STA+EMENT
1910 IF VO < 2 THEN GOSUB 2660
1920 IF VO = 2 THEN GOSUB 3140
1930 '
1940 GOTO 1840
1950 PRINT
1960 PRINT "   TEST IS FINISHED --- PRESS RETURN TO CONTINUE "
1970 A$=INKEY$ : IF A$=""  THEN 1970
1980 CLS:SCREEN 1:COLOR 8,0: LINE (0,0)-(319,199),1,B
```

```
1990 LOCATE 10,7 :PRINT"SAVE DATA IN DISKETTE <Y,N> "
2000 A$=INKEY$: IF A$="" THEN 2000
2010 IF A$="Y" OR A$="y" THEN 2030
2020 IF A$ <> "Y" OR A$ <> "y" THEN 2300
2030 LOCATE 13,7:PRINT"INSERT DISKETTE IN DRIVE A  "
2040 LOCATE 15,12:PRINT"--PRESS RETURN-- "
2050 A$=INKEY$ :IF A$="" THEN 2050
2060 ' CONVERT STRAIN GAGE OUTPUT TO MICRO-STRAIN
2070 FOR J = 0 TO II
2080 SG(J)=SAG(J)*MN2
2090 NEXT
2100 '

2110 OPEN "O",1,ID$
2120 PRINT#1,"  FILE NAME...";ID$;" DATE.....";DA$;" TYPE......";TY$
2130 PRINT#1,
2140 PRINT#1,"            SPECIMEN DIMENSION (ROUND BAR) "
2150 PRINT#1,
2160 PRINT#1,"  INITIAL DIAMETER (in.).......=";ID
2170 PRINT#1,"  GAGE LENGTH (in.)............=";IL
2180 PRINT#1,"  STRAIN RATE LOADING (/s).....=";SR1
2190 PRINT#1,"  STRAIN RATE UNLOADING (/s)...=";SR2
2200 PRINT#1,"  NO. OF CONVERSIONS/SEC.......=";NOC
2210 PRINT#1,"  LOAD FACTOR..................=";NO
2220 PRINT#1,"  EXETENSOMETER FACTOR (A/D)...=";N1
2230 PRINT#1,"  STRAIN GAGE FACTOR (E-06)....=";MN2
2240 PRINT#1,
2250 PRINT#1,"   II   LOAD  EXTEN.    SG1  "
2260 FOR J=0 TO II
2270 PRINT#1,USING"  #### ###### ##.#### ######";J,LC(J),EX(J),SG(J)
2280 NEXT
2290 CLOSE #1
2300 WIDTH 80 :SCREEN 0
2310 NN=5
2320 PRINT:INPUT" Do You Wish To Continue < Y,N > ";B$
2330 IF B$="Y" OR B$="y" THEN 1840
2340 END
2350 '
2360 '*********** SUBROUTINE TO CALIBRATE STRAIN GAGES ***************
2370 '
2380 CLS: LINE (0,0)-(319,199),1,B
2390 LOCATE 3,6 :PRINT"Channel =               ";ACH
2400 LOCATE 5,5 :INPUT"Gage Resistance (ohms) ";GR
2410 LOCATE 8,5 :INPUT"No. of Active Gages    ";N
2420 LOCATE 11,5 :INPUT"Gage factor            ";GF
2430 LOCATE 14,5 :INPUT"Shunt Resistance (ohms)";SR
2440 ASTN = GR/(N*GF*SR)
2450 CLS: LINE (0,0)-(319,199),1,B
2460 LOCATE 5,5 :PRINT"CHANNEL                ";ACH
2470 LOCATE 10,5 :PRINT"< Plug in Shunt resistace >"
2480 LOCATE 21,9 :PRINT"PRESS RETURN TO CONTINUE "
2490 A$=INKEY$ : IF A$="" THEN 2490
```

```
2500 MD%=3
2510 CALL AI016 (MD%,DIO%(0),FLAG%)
2520 CLS: LINE (0,0)-(319,199),1,B
2530 LOCATE 7,5 :PRINT USING "##.#### STRAIN = ##### COUNTS";ASTN,DIO%(0)
2540 N2=(ASTN/DIO%(0))*1000000! :' Strain Gage Factor For Gage 1
2550 MN2=-N2
2560 Z2=MN2/NOC
2570 LOCATE 10,17 :PRINT"  OR "
2580 LOCATE 13,5 :PRINT USING " 1 C = ##.#### MICRO STRAIN ";MN2
2590 LOCATE 20,7 :PRINT"< Remove Shunt Resistance >"
2600 LOCATE 23,9 :PRINT "Press Return to Continue "
2610 A$=INKEY$ : IF A$="" THEN 2610
2620 RETURN
2630 '**************** END OF SUBROUTINE ********************************
2640 '

2650 '******* SUBROUTINE USED TO APPLY A COMMAND **********************
2660 '
2670 IF VO=0 THEN XX%=XX%+4 :'XX%=XX%+(MUL1/(IL+EX(II))) :' LOADING
INCREMENT
2680 IF VO=1 THEN XX%=XX%-4 :'XX%=XX%-(MUU1/(IL+EX(II))) :' UNLOADING
INCREMENT
2690 MD%=15 :' APPLY DISPLACEMENT USING D/A 0
2700 DIO%(0)=0 : ' D/A USING CHANNEL 0
2710 DIO%(1)=XX% : ' D/A DATA  (ZERO COUNT)
2720 T$=RIGHT$(TIME$,1)
2730 IF T$ < > S$ THEN 2750
2740 GOTO 2720
2750 CALL AI016 (MD%,DIO%(0),FLAG%)
2760 IF FLAG% <> 0 THEN PRINT " OUTPUT DATA (D/A#0) ERROR  "
2770 '
2780 S$=T$:LOCATE 25, 72:PRINT PT
2790 '
2800 MD%=3 : ' READ CHANNELS 0 - 2
2810 A0=0: A1=0 :A2=0
2820 K0=0: K1=0 :K2=0
2830 FOR K=1 TO NOC
2840 FOR J=0 TO 2
2850 '
2860 CALL AI016 (MD%,DIO%(0),FLAG%)
2870 '
2880 IF FLAG% <> 0 THEN PRINT " A/D CONVERSION ERROR  "
2890 IF DIO%(1)=0 THEN K0=DIO%(0)
2900 IF DIO%(1)=1 THEN K1=DIO%(0)
2910 IF DIO%(1)=2 THEN K2=DIO%(0)
2920 NEXT J
2930 '
2940 A0=A0+K0 :A1=A1+K1 :A2=A2+K2
2950 NEXT K
2960 '
2970 IF NNN=1 THEN 3000
2980 IF PT < > I+1 THEN RETURN
```

```
2990 I=PT
3000 II=II+1
3010 LC(II)=A0*Z0
3020 EX(II)=A1*Z1
3030 SAG(II)=A2/NOC
3040 LOCATE 24,1
3050 PRINT USING"     ####   ####   ######   ##.####
######";PT,II,LC(II),EX(II),SAG(II);
3060 PRINT USING " ####";XX%;
3070 LPRINT USING"     ####   ####   ######   ##.####
######";PT,II,LC(II),EX(II),SAG(II);
3080 LPRINT USING " ####";XX%
3090 PRINT "    "T$"  "S$
3100 '
3110 RETURN
3120 '*******************  END OF SUBROUTINE  ****************************
3130 '

3140 '******* SUBROUTINE USED TO MEASURE A/D OUTPUTS ONLY ****************
3150 NNN=1
3160 T$=RIGHT$(TIME$,1)
3170 IF T$ < > S$ THEN 3190
3180 GOTO 3160
3190 '
3200 S$=T$:LOCATE 25, 72:PRINT PT
3210 '
3220 MD%=3 : ' READ CHANNELS 0 - 2
3230 A0=0: A1=0 :A2=0
3240 K0=0: K1=0 :K2=0
3250 FOR K=1 TO NOC
3260 FOR J=0 TO 2
3270 '
3280 CALL AIO16 (MD%,DIO%(0),FLAG%)
3290 '
3300 IF FLAG% <> 0 THEN PRINT " A/D CONVERSION ERROR  "
3310 IF DIO%(1)=0 THEN K0=DIO%(0)
3320 IF DIO%(1)=1 THEN K1=DIO%(0)
3330 IF DIO%(1)=2 THEN K2=DIO%(0)
3340 NEXT J
3350 '
3360 A0=A0+K0 :A1=A1+K1 :A2=A2+K2
3370 NEXT K
3380 '
3390 IF PT < > I+15 THEN RETURN
3400 I=PT
3410 II=II+1
3420 LC(II)=A0*Z0
3430 EX(II)=A1*Z1
3440 SAG(II)=A2/NOC
3450 LOCATE 24,1
3460 PRINT USING"     ####   ####   ######   ##.####
######";PT,II,LC(II),EX(II),SAG(II);
```

```
3470 PRINT USING " ####";XX%;
3480   LPRINT   USING"      ####    ####    ######    ##.####
######";PT,II,LC(II),EX(II),SAG(II);
3490 LPRINT USING " ####";XX%
3500 PRINT "    "T$"  "S$
3510 '
3520 RETURN
```

Appendix D

```
10 '**********************************************************************
20 '                                                                     *
30 '  THIS PROGRAM CONTROLS THE MTS TESTING SYSTEM WITH METRABYTE DATA    *
40 '                                                                     *
50 '  ACQUISITION BOARD. THE TEST IS DONE FOR THE BENDING OF PLATE.       *
60 '                                                                     *
70 '                                                                     *
80 '  DEVELOPED BY : LOUAY N. MOHAMMAD                                   *
90 '  SUPERVISOR   : DR. GEORGE Z. VOYIADJIS                             *
100 ' DEVELOPED AT : LOUISIANA STATE UNIVERSITY                          *
120 '                                                                    *
130 '             CHANNELS USED ----- 0 ........ LOAD CELL               *
140 '                                 1 ........ LVDT (Center Deflection) *
150 '                                 2 ........ STRAIN GAGE 1 (SG1)     *
160 '                                 3 ........ STRAIN GAGE 2           *
170 '                                 4 ........ STRAIN GAGE 3           *
180 '                                 5 ........ STRAIN GAGE 4           *
190 '                                 8 ........ ACTUATOR                *
200 '                                                                    *
210 '*********************************************************************
220 '
230 '
240 ' DEFINE WORK SPACE & INITIALIZE THE BOARD
250 '
260 '
270 CLEAR, 49152!
280 SCREEN 0,0,0:KEY OFF:CLS
290 DEF SEG = 0
300 SG = 256 * PEEK(&H511) + PEEK(&H510)
310 SG = SG + 49152!/16
320 DEF SEG = SG
330 BLOAD "AIO16.BIN", 0 :' Load binary files
340 DIM DIO%(8)
350 DIO%(0)=768 : DIO%(1) = 2 : DIO%(2) = 3
360 DASH = 0
370 FLAG% = 0
380 MD% = 0
390 CALL AIO16 (MD%, DIO%(0),FLAG%)
400 DIM LC(1000),LVDT(1000),SG1(1000),SG2(1000),SG3(1000),SG4(1000)
410 '
420 '************** CALIBRATIONS FACTORS **********************
430 '
440 CLS:SCREEN 1:COLOR 8,0: LINE (0,0)-(319,199),1,B
450 LOCATE 3,13:PRINT"TEST INFORMATION"
460 LOCATE 9,2: INPUT"No. of Conversions / sec ( > 0 )";NOC
470 IF NOC=0 THEN 440
480 Z=1!/NOC
490 C0=1.0069571#:C1=1! :C8=1/.989
```

```
500 NO=(50000! * CO )/2048 :' LOAD FACTOR
510 N1=(.25 * C1 )/710 :' LVDT FACTOR (Center Deflection)
520 N8=(2.2 * C8 )/900 :' LVDT FACTOR (Actuator)
530 ZO=Z*NO : Z1=Z*N1 :Z8=Z*N8
540 '
550 '****************** TEST INFORMATION ***********************
560 '
570 CLS:SCREEN 1:COLOR 8,0: LINE (0,0)-(319,199),1,B
580 LOCATE 2,13:PRINT"TEST INFORMATION"
590 LOCATE 5,2: INPUT"TEST ID                      ";ID$
600 LOCATE 8,2: INPUT"DATE OF TEST                 ";DA$
610 LOCATE 11,2: INPUT"TYPE OF SPECIMEN            ";TY$
620 LOCATE 14,2: INPUT"STRAIN RATE (LOADING)       ";SR1$
630 LOCATE 17,2: INPUT"PLATE LENGTH                ";PLTL
640 LOCATE 20,2: INPUT"PLATE WIDTH                 ";PLTW
650 LOCATE 23,2: INPUT"PLATE THICKNESS             ";PLTT
660 '
670 ' SELECT PROPER CHANNEL FOR CALIBRATION
680 '
690 CLS: LINE (0,0)-(319,199),1,B
700 LOCATE 5,3 :PRINT " CHANNEL 0 = LOAD CELL  "
710 LOCATE 7,3 :PRINT " CHANNEL 1 = LVDT (deflection)"
720 LOCATE 9,3 :PRINT " CHANNEL 2 = STRAIN GAGE 1 "
730 LOCATE 11,3 :PRINT " CHANNEL 3 = STRAIN GAGE 2 "
740 LOCATE 13,3 :PRINT " CHANNEL 4 = STRAIN GAGE 3 "
750 LOCATE 15,3 :PRINT " CHANNEL 5 = STRAIN GAGE 4 "
760 LOCATE 17,3 :PRINT " CHANNEL 6 = STRAIN GAGE 5 "
770 LOCATE 19,3 :PRINT " CHANNEL 7 = STRAIN GAGE 6 "
780 LOCATE 21,3 :PRINT " CHANNEL 8 = LVDT (actuator) "
790 LOCATE 23,2 :PRINT "SELECT CHANNEL TO MONITOR (0 - 8) "
800 A$=INKEY$ :IF A$="" THEN 800
810 '
820 ' A$ is in the range of 0 - 9
830 '
840 IF ASC(A$) >47 AND ASC(A$) < 58 THEN ACH=ASC(A$) - 48
850 CLS:LINE (0,0)-(319,199),1,B
860 LOCATE 2,5 :PRINT"CHANNEL       = "
870 LOCATE 4,5 :PRINT"LOAD          = "
880 LOCATE 6,5 :PRINT"LVDT (defl.) = "
890 LOCATE 8,5 :PRINT"STRAIN G1     = "
900 LOCATE 10,5 :PRINT"STRAIN G2     = "
910 LOCATE 12,5 :PRINT"STRAIN G3     = "
920 LOCATE 14,5 :PRINT"STRAIN G4     = "
930 LOCATE 16,5 :PRINT"STRAIN G5     = "
940 LOCATE 18,5 :PRINT"STRAIN G6     = "
950 LOCATE 20,5 :PRINT"LVDT (actu.) = "
960 LOCATE 22,11 :PRINT "Press ESC To Finish "
970 LOCATE 23,5  :PRINT"Press S to Select Another Channel "
980 '
990 ' SET MULTIPLEXER SCAN LIMIT
1000 '
1010 MD%=1
```

```
1020 DIO%(0)=ACH :'LOWER SCAN LIMIT
1030 DIO%(1)=ACH :'UPPER SCAN LIMIT
1040 CALL AIO16 (MD%,DIO%(0),FLAG%)
1050 IF FLAG% <> 0 THEN PRINT " MULTIPLEXER SCAN LIMIT ERROR "
1060 '
1070 ' PERFORM A/D CONVERSION USING MODE 3
1080 '
1090 MD%=3
1100 CALL AIO16 (MD%,DIO%(0),FLAG%)
1110 IF FLAG% <> 0 THEN PRINT " A/D CONVERSION ERROR  "
1120 LOCATE 2,20 :PRINT DIO%(1)
1130 IF DIO%(1)=0 THEN OFST0%=DIO%(0)
1140 IF DIO%(1)=0 THEN LOCATE 4,20 :PRINT DIO%(0)
1150 IF DIO%(1)=1 THEN OFST1%=DIO%(0)
1160 IF DIO%(1)=1 THEN LOCATE 6,20 :PRINT DIO%(0)
1170 IF DIO%(1)=2 THEN LOCATE 8,20 :PRINT DIO%(0)
1180 IF DIO%(1)=3 THEN LOCATE 10,20 :PRINT DIO%(0)
1190 IF DIO%(1)=4 THEN LOCATE 12,20 :PRINT DIO%(0)
1200 IF DIO%(1)=5 THEN LOCATE 14,20 :PRINT DIO%(0)
1210 IF DIO%(1)=6 THEN LOCATE 16,20 :PRINT DIO%(0)
1220 IF DIO%(1)=7 THEN LOCATE 18,20 :PRINT DIO%(0)
1230 IF DIO%(1)=8 THEN LOCATE 20,20 :PRINT DIO%(0)
1240 A$=INKEY$ : IF A$=CHR$(27) THEN 1270
1250 IF A$="S" OR A$="s" THEN 690
1260 GOTO 1090
1270 IF ACH < 2 OR ACH > 7 THEN 1340
1280 GOSUB 2350
1290 CLS:LINE (0,0)-(319,199),1,B
1300 LOCATE 13,3:PRINT"Calibrate Another Strain Gage <Y,N> "
1310 A$=INKEY$: IF A$="" THEN 1310
1320 IF A$ = "Y" OR A$ = "y" THEN 690
1330 IF A$ <> "N" AND A$ <> "n" THEN 1310
1340 CLS: SCREEN 0 : WIDTH 80
1350 PT=0 : XX=4095 :' Used only for plate bending
1360 I=0 : II=0
1370 T$="0" : S$="0" :' Used for timing
1380 '
1390 CLS:PRINT
1400 PRINT "  TEST ID               : ";ID$
1410 PRINT "  DATE OF TEST          : ";DA$
1420 PRINT "  TYPE OF SPECIMEN      : ";TY$
1430 PRINT "  STRAIN RATE (LOADING) : ";SR1$
1440 PRINT "  PLATE LENGTH          : ";PLTL
1450 PRINT "  PLATE WIDTH           : ";PLTW
1460 PRINT "  PLATE THICKNESS       : ";PLTT
1470 PRINT
1480 PRINT"   PT   II LOAD  LVDT   SG1   SG2   SG3   SG4   XX "
1490 '
1500 '   SET MULTIPLEXER TO SCAN CHANNELS 0 - 5
1510 '
1520 MD%=1
1530 DIO%(0)=0
```

```
1540 DIO%(1)=5
1550 CALL AIO16 (MD%,DIO%(0),FLAG%)
1560 IF FLAG% <> 0 THEN PRINT " MULTIPLEXER SCAN LIMIT ERROR "


1570 '
1580 ' INITIALIZE (I.E.,ZERO INPUT (DISPLACEMENT) TO DA#0 )
1590 '
1600 '                    USING MODE 15
1610 '
1620 GOSUB 2730 :' APPLY ZERO COMMAND & READ THE SET CHANNELS BY MODE 1
1630 LC(II)=A0/NOC:LVDT(II)=A1/NOC
1640 SG1(II)=A2/NOC:SG2(II)=A3/NOC:SG3(II)=A4/NOC
1650 SG4(II)=A5/NOC
1660 LPRINT USING"#### #### ##### ##### #####
#####";PT,II,LC(II),LVDT(II),SG1(II),SG2(II);
1670 LPRINT USING" ##### ##### ####";SG3(II),SG4(II),XX
1680 PRINT USING"#### #### ##### ##### #####
#####";PT,II,LC(II),LVDT(II),SG1(II),SG2(II);
1690 PRINT USING" ##### ##### ####";SG3(II),SG4(II),XX;
1700 PRINT "   "T$"   "S$
1710 PRINT " HIT ANY KEY TO START "
1720 A$=INKEY$ :IF A$="" THEN 1720
1730 '
1740 ' APPLY DISPLACEMENT USING DA#0 ----MODE 15----
1750 '
1760 '
1770 LOCATE 25,3:PRINT"ENTER  8 TO LOAD        2 TO UNLOAD              ESC TO
ABORT
1780 LOCATE 13,1
1790 TIME$="0:0:0"
1800 A$=INKEY$:IF A$=CHR$(56) THEN VO=0 :' CHR$(56)=NO. 8
1810 IF A$=CHR$(50) THEN VO=1 :' CHR$(50)=NO. 2
1820 IF A$=CHR$(27) THEN 1920
1830 'IF VO=0 THEN XX=XX+98
1840 'IF VO=1 THEN XX=XX-98
1850 'IF PT = 300 THEN 1750
1860 '
1870 PT=PT+1:' XX=XX-1
1880 IF VO=0 THEN GOSUB 2710
1890 IF VO=1 THEN GOSUB 3200
1900 '
1910 GOTO  1800
1920 PRINT
1930 GOTO 1950
1940 PRINT "  PT EXCEEDED DIMENSION STATEMENT "
1950 M=2*PT
1960 PRINT "   TEST IS FINISHED --- PRESS RETURN TO CONTINUE "
1970 A$=INKEY$ : IF A$=""  THEN 1970
1980 CLS:SCREEN 1:COLOR 8,0: LINE (0,0)-(319,199),1,B
1990 LOCATE 10,7 :PRINT"SAVE DATA IN DISKETTE <Y,N> "
2000 A$=INKEY$: IF A$="" THEN 2000
```

```
2010 IF A$="Y" OR A$="y" THEN 2030
2020 IF A$ <> "Y" OR A$ <> "y" THEN SCREEN 0:WIDTH 80:END
2030 LOCATE 13,7:PRINT"INSERT DISKETTE IN DRIVE A  "
2040 LOCATE 15,12:PRINT"--PRESS RETURN-- "
2050 A$=INKEY$ :IF A$="" THEN 2050
2060 MN2=-N2
2070 MN3=-N3
2080 MN4=-N4
2090 MN5=-N5
2100 OPEN "O",1,ID$
2110  PRINT#1," File  name  is  ";ID$;"  Date:";DA$;"  Type:";TY$;"  Strain
rate:";SR1$
2120 PRINT#1,
2130 PRINT#1,"                              Plate Dimension"
2140 PRINT#1,
2150 PRINT#1," Length (IN.)=";PLTL;" Width (IN.)=";PLTW;"Thickness (IN.)=";PLTT
2160 PRINT#1,
2170 PRINT#1," No. of conversions / sec = ";NOC
2180 PRINT#1," Load factor ............ = ";N0
2190 PRINT#1," LVDT factor ............ = ";N1
2200 PRINT#1," SG 1 factor ............ = ";MN2
2210 PRINT#1," SG 2 factor ............ = ";MN3
2220 PRINT#1," SG 3 factor ............ = ";MN4
2230 PRINT#1," SG 4 factor ............ = ";MN5
2240 PRINT#1,
2250 PRINT#1,"   II LOAD  LVDT   SG1   SG2   SG3   SG4   "
2260 PRINT#1,
2270 FOR J=0 TO II
2280  PRINT#1,  USING"####   #####   #####   #####
#####";J,LC(J),LVDT(J),SG1(J),SG2(J);
2290 PRINT#1, USING" ##### ##### ";SG3(J),SG4(J)
2300 NEXT
2310 CLOSE #1
2320 SCREEN 0:WIDTH 80 :END
2330 '
2340 '*********** SUBROUTINE TO CALIBRATE STRAIN GAGES ***************
2350 ASIN=0:B=0
2360 '
2370 CLS: LINE (0,0)-(319,199),1,B
2380 LOCATE 3,6 :PRINT"Channel =                ";ACH
2390 LOCATE 5,5 :INPUT"Gage Resistance (ohms) ";GR
2400 LOCATE 8,5 :INPUT"No. of Active Gages    ";N
2410 LOCATE 11,5 :INPUT"Gage factor            ";GF
2420 LOCATE 14,5 :INPUT"Shunt Resistance (ohms)";SR
2430 ASIN = GR/(N*GF*SR)
2440 CLS: LINE (0,0)-(319,199),1,B
2450 LOCATE 5,5 :PRINT"CHANNEL                  ";ACH
2460 LOCATE 10,5 :PRINT"< Plug in Shunt resistace >"
2470 LOCATE 21,9 :PRINT"PRESS RETURN TO CONTINUE "
2480 A$=INKEY$ : IF A$="" THEN 2480
2490 MD%=3
2500 CALL AIO16 (MD%,DIO%(0),FLAG%)
```

```
2510 CLS: LINE (0,0)-(319,199),1,B
2520 LOCATE 7,5 :PRINT USING "##.#### STRAIN = ##### COUNTS";ASTN,DIO%(0)
2530 IF ACH=2 THEN N2=(ASTN/DIO%(0))*1000000! :' Strain Gage Factor For Gage 1
2540 IF ACH=3 THEN N3=(ASTN/DIO%(0))*1000000! :' Strain Gage Factor For Gage 2
2550 IF ACH=4 THEN N4=(ASTN/DIO%(0))*1000000! :' Strain Gage Factor For Gage 3
2560 IF ACH=5 THEN N5=(ASTN/DIO%(0))*1000000! :' Strain Gage Factor For Gage 4
2570 IF ACH=2 THEN B=N2
2580 IF ACH=3 THEN B=N3
2590 IF ACH=4 THEN B=N4
2600 IF ACH=5 THEN B=N5
2610 B1=ABS(B)
2620 LOCATE 10,17 :PRINT"  OR "
2630 LOCATE 13,5 :PRINT USING " 1 C = ##.#### MICRO STRAIN ";B1
2640 LOCATE 20,7 :PRINT"< Remove Shunt Resistance >"
2650 LOCATE 23,9 :PRINT "Press Return to Continue "
2660 A$=INKEY$ : IF A$="" THEN 2660
2670 RETURN
2680 '***************** END OF SUBROUTINE ********************************
2690 '
2700 '******  SUBROUTINE  THAT APPLY COMMAND USING DA#0  ***********
2710 '
2720 XX=XX-5
2730 MD%=15 :' APPLY COMMAND SIGNAL USING D/A 0
2740 DIO%(0)=0 : ' D/A USING CHANNEL 0
2750 DIO%(1)=XX : ' D/A DATA (0-4095 count)
2760 T$=RIGHT$(TIME$,1)
2770 IF T$ < > S$ THEN 2790
2780 GOTO 2760
2790 CALL AIO16 (MD%,DIO%(0),FLAG%)
2800 '
2810 S$=T$
2820 '
2830 A0=0:A1=0:A2=0:A3=0:A4=0
2840 A5=0
2850 K0=0:K1=0:K2=0:K3=0:K4=0
2860 K5=0
2870 MD%=3 : ' READ CHANNELS 0 - 5
2880 FOR K=1 TO NOC :'no. of conversions for each channel
2890 FOR J=0 TO 5
2900 '
2910 CALL AIO16 (MD%,DIO%(0),FLAG%)
2920 '
2930 IF FLAG% <> 0 THEN PRINT " A/D CONVERSION ERROR  "
2940 IF DIO%(1)=0 THEN K0=DIO%(0)-OFST0%
2950 IF DIO%(1)=1 THEN K1=DIO%(0)-OFST1%
2960 IF DIO%(1)=2 THEN K2=DIO%(0)
2970 IF DIO%(1)=3 THEN K3=DIO%(0)
2980 IF DIO%(1)=4 THEN K4=DIO%(0)
2990 IF DIO%(1)=5 THEN K5=DIO%(0)
3000 NEXT J
3010 A0=A0+K0:A1=A1+K1:A2=A2+K2:A3=A3+K3:A4=A4+K4
3020 A5=A5+K5
```

```
3030 NEXT K
3040 IF NNN=1 THEN 3070
3050 IF PT < > I+1 THEN RETURN
3060 I=PT
3070 II=II+1
3080 LC(II)=A0/NOC:LVDT(II)=A1/NOC
3090 SG1(II)=A2/NOC:SG2(II)=A3/NOC:SG3(II)=A4/NOC
3100 SG4(II)=A5/NOC
3110 LPRINT USING"#### #### ##### ##### #####
#####";PT,II,LC(II),LVDT(II),SG1(II),SG2(II);
3120 LPRINT USING" ##### ##### ####";SG3(II),SG4(II),XX
3130 PRINT USING"#### #### ##### ##### #####
#####";PT,II,LC(II),LVDT(II),SG1(II),SG2(II);
3140 PRINT USING" ##### ##### ####";SG3(II),SG4(II),XX;
3150 PRINT "   "T$"   "S$
3160 '
3170 RETURN
3180 '***************   END OF SUBROUTINE   ******************************
3190 '
3200 '******* SUBROUTINE USED TO MEASURE A/D OUTPUTS ONLY ***************
3210 NNN=1
3220 T$=RIGHT$(TIME$,1)
3230 IF T$ < > S$ THEN 3250
3240 GOTO 3220
3250 '
3260 S$=T$
3270 '
3280 A0=0:A1=0:A2=0:A3=0:A4=0
3290 A5=0
3300 K0=0:K1=0:K2=0:K3=0:K4=0
3310 K5=0
3320 MD%=3 : ' READ CHANNELS 0 - 5
3330 FOR K=1 TO NOC :'no. of conversions for each channel
3340 FOR J=0 TO 5
3350 '
3360 CALL AIO16 (MD%,DIO%(0),FLAG%)
3370 '
3380 IF FLAG% <> 0 THEN PRINT " A/D CONVERSION ERROR  "
3390 IF DIO%(1)=0 THEN K0=DIO%(0)-OFST0%
3400 IF DIO%(1)=1 THEN K1=DIO%(0)-OFST1%
3410 IF DIO%(1)=2 THEN K2=DIO%(0)
3420 IF DIO%(1)=3 THEN K3=DIO%(0)
3430 IF DIO%(1)=4 THEN K4=DIO%(0)
3440 IF DIO%(1)=5 THEN K5=DIO%(0)
3450 NEXT J
3460 A0=A0+K0:A1=A1+K1:A2=A2+K2:A3=A3+K3:A4=A4+K4
3470 A5=A5+K5
3480 NEXT K
3490 IF PT < > I+15 THEN RETURN
3500 II=II+1
3510 I = PT
3520 LC(II)=A0/NOC:LVDT(II)=A1/NOC
```

```
3530 SG1(II)=A2/NOC:SG2(II)=A3/NOC:SG3(II)=A4/NOC
3540 SG4(II)=A5/NOC
3550 LPRINT USING"#### #### ##### ##### #####
#####";PT,II,LC(II),LVDT(II),SG1(II),SG2(II);
3560 LPRINT USING" ##### ##### ####";SG3(II),SG4(II),XX
3570 PRINT USING"#### #### ##### ##### #####
#####";PT,II,LC(II),LVDT(II),SG1(II),SG2(II);
3580 PRINT USING" ##### ##### ####";SG3(II),SG4(II),XX;
3590 PRINT "   "T$"   "S$
3600 '
3610 RETURN
```

VITA

Louay Nadhim Mohammad was born on July 30, 1957, in Baghdad, Iraq. He enrolled in the Civil Engineering curriculum at Louisiana State University, Baton Rouge, in August 1977. In May 1980 he received his Bachelor of Science degree in Civil Engineering.

He enrolled in the Graduate School at Louisiana State University in August 1980. He was awarded a Graduate Teaching Assistantship in January 1981. In August 1982 he received his Master of Science degree in Civil Engineering, In September 1987, he became a Research Associate in the Louisiana Transportation Research Center. He is now a candidate for the degree of Doctor of Philosophy in Civil Engineering.

DOCTORAL EXAMINATION AND DISSERTATION REPORT

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Title of Dissertation: Constitutive Equations for Large Deformation, Rate Dependent Plasticity for Metal Alloys

Approved:

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Date of Examination:

5/18/89